Discussion Papers In Economics And Business

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Discussion Paper 26-04

April 2026

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Education Financing under Borrowing Constraints: Political Economy Implications for Intergenerational Fiscal Policy and Growth*

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April 3, 2026

Abstract

This paper examines how borrowing constraints faced by young individuals shape the political allocation of public spending between pensions and education. We develop a three-period overlappinggenerations model with human capital formation in which individuals finance tertiary education privately through credit markets subject to borrowing limits. Fiscal policy—comprising pension benefits, non-tertiary education expenditures, public funding of tertiary education, and the labor income tax rate—is determined through probabilistic voting. Calibrating the model to Japan and the United States, we find that relaxing borrowing constraints lowers pension spending and the share of public funding for tertiary education while increasing tertiary education spending and economic growth; the response of non-tertiary education spending is monotonic in Japan but hump-shaped in the United States. Based on projected declines in population growth, both countries are predicted to experience increases in pension spending and in the share of public funding for tertiary education, as well as decreases in spending on both non-tertiary and tertiary education and slower economic growth.

- Keywords: Borrowing constraint; Non-tertiary and tertiary education; Pensions; Probabilistic voting; Economic growth
- JEL Classification: E60, H52, I22, J11, O40

*The authors gratefully acknowledge financial support from Grant-in-Aid for Early-Career Scientists from the Japan Society for the Promotion of Science (No. 24K16388, Uchida) and Grant-in-Aid for Scientific Research (C) from the Japan Society for the Promotion of Science (No. 24K04926, Ono).

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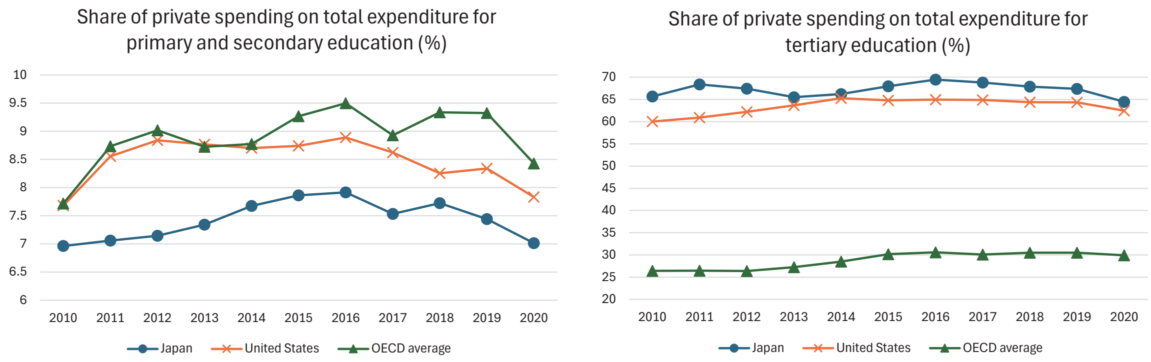
1 Introduction

Declining birth rates and the associated slowdown in population growth have been observed in most member countries of the Organization for Economic Cooperation and Development (OECD) in recent decades. This demographic trend leads to an increasing share of older adults in the voting population (OECD, 2016), and is projected to persist over the coming decades (Rouzet et al., 2019). As a consequence, each working-age individual is expected to bear a growing fiscal burden to support pension benefits for older adults, while government spending on programs that do not directly benefit this demographic—such as public education—may decline (Gonzalez-Eiras and Niepelt, 2012). However, older adults may not necessarily oppose education spending, as it can enhance productivity and thereby generate higher fiscal revenues to support future pension provision (Gradstein and Kaganovich, 2004). These conflicting incentives raise the issue of how governments allocate their limited budgets between pensions for older adults and education for younger generations in the context of declining population growth rates.

Recent studies on this issue include Gonzalez-Eiras and Niepelt (2012), Lancia and Russo (2016), Ono and Uchida (2016), Bishnu and Wang (2017), and Uchida and Ono (2024b). These studies overlook the role of borrowing constraints faced by the young in financing their education. Specifically, these studies either assume the absence of private financing of education (Gonzalez-Eiras and Niepelt, 2012; Ono and Uchida, 2016; Uchida and Ono, 2024b) or treat private and public education investments as perfect substitutes, resulting in complete crowding-out of private financing by public provision (Lancia and Russo, 2016; Bishnu and Wang, 2017). Such assumptions may hold for many European countries where the private share of tertiary education costs is low and tuition fees are either waived or kept at minimal levels (OECD, 2024). However, these assumptions do not apply to Japan and the United States, where the private share is comparatively high, as illustrated in Figure 1. In these countries, private expenditures for tertiary education—particularly tuition fees—are substantial, making it common for individuals to finance their tertiary education expenditures through education loan markets. Nevertheless, borrowing constraints often bind, preventing individuals from borrowing sufficiently to cover these expenditures (Soares, 2003; Avery and Turner, 2012; Diris and Ooghe, 2018).

Several early studies have examined the impact of borrowing constraints on policy preferences and the resulting economic outcomes. Among the earliest contributions are Fernandez and Rogerson (1995, 1996), who analyze settings in which individuals face borrowing constraints that prevent them from financing education expenditures through future earnings. They show that such borrowing constraints are central to the political and economic consequences of education systems. Building on this insight, Soares (2003) develops a political economy model in which support for education funding arises from self-interest under borrowing constraints. However, his model abstracts from intergenerational conflict over multiple policy instruments due to its exclusive focus on education policy. In contrast, Bellettini and Ceroni (1999) abstract from education policy and instead focus on redistribution through lump-sum income transfers, and



(a) (b)

Figure 1: The shares of private spending in total expenditure for (a) primary and secondary education, and (b) tertiary education.
Note. During the period 2010–2012, Japan and the United States ranked first and second, respectively, among OECD member countries in terms of the share in Panel (b), and since 2013, both countries have consistently ranked within the top two to four.
Source: The United Nations World Population Prospects (see Appendix E for data sources).

therefore remain limited to a single policy instrument. Camargo and Stein (2022) partially overcome this limitation by introducing multiple policy instruments (education and transfers), but their simplified two-period framework still does not fully account for intergenerational tensions or the long-run effects on economic growth.

The present study overcomes the limitations of the aforementioned studies by examining the following three questions: (i) How does the strength of borrowing constraints influence the government's intergenerational allocation of spending between pensions and education, and what are the implications for economic growth? (ii) When partial public funding of tertiary education is introduced to alleviate borrowing constraints, how are economic growth and intergenerational resource redistribution affected? (iii) In advanced economies with well-developed education loan markets, how do two key factors—the relaxation of borrowing constraints faced by the young and the rising population share of older adults—affect public expenditures on pensions and education, and how do they influence economic growth? Through this analysis, the paper argues that borrowing constraints play a pivotal role in shaping the future trajectory of intergenerational redistribution policies—namely, pensions and education—and their implications for economic growth in the context of declining population growth.

To investigate these questions, this study employs a three-period overlapping-generations model with physical and human capital (see, e.g., Boldrin and Montes, 2005; Docquier et al., 2007; Del Rey and Lopez-Garcia, 2013, 2017). In the model, individuals in each generation live through three life stages: youth, middle age, and older adult age. During youth, individuals borrow in credit markets to finance private expenditures on tertiary education against their labor income in middle age, thereby enhancing their labor productivity during that stage of life. Due to borrowing constraints, individuals are unable to raise sufficient funds during youth to fully finance their desired expenditures, but the constraints are relaxed by partial public funding of tertiary education. In middle age, individuals choose labor supply and allocate their after-tax income between consumption and savings. In older adult age, they receive returns on their savings as well as pension benefits, both of which are used for consumption. Government spending, consisting of pay-as-you-go pension benefits, full funding of non-tertiary education, and partial funding of tertiary education, is financed through a labor income tax on the middle-aged and is determined each period through probabilistic voting (Lindbeck and Weibull, 1987; Persson and Tabellini, 2002), which maximizes the political objective function, that is, the weighted sum of generational utilities.

Within the above framework, we first derive the policy functions for pensions and non-tertiary education expenditures, the portion of public funding of tertiary education, and the labor income tax rate. We find that in each period, the government—representing the middle-aged and older adult generations—selects these policy instruments such that the marginal utility of each type of expenditure, per unit increase in spending, is equalized to the marginal disutility of labor income taxation, per unit increase in tax revenue, referred to as the marginal cost of taxation (MCT). Based on this characterization, we find that the ratios of pension and non-tertiary education expenditures to GDP, the portion of public funding of tertiary education, and the labor income tax rate remain constant over time and depend solely on structural parameters, such as the degree of borrowing constraints and the population growth rate.

Based on the derivation of the policy functions, we first focus on the case of fully private funding of tertiary education, for which qualitative comparative statics analysis is tractable, and examine how a relaxation of the borrowing constraint–interpretable as resulting from institutional or legal changes–affects the policy functions. A relaxation of the borrowing constraint increases private tertiary education expenditures during youth. This, in turn, raises savings in older adult age through asset market equilibrium and strengthens the perceived tax burden on older adults, as the associated decline in labor supply reduces the interest rate. Furthermore, higher private tertiary education spending increases loan repayments and thereby reduces net after-tax income during middle age. Consequently, MCT increases, leading to a lower labor income tax rate.

With respect to the pension expenditure-to-GDP ratio, increased savings reduce the need for pension benefits, leading to a decline in their marginal benefit and, ultimately, a reduction in pension spending. Regarding the non-tertiary education expenditure-to-GDP ratio, the effect of relaxing the borrowing constraint depends critically on the relative weight of the older adults in the government's objective function. When the relative weight is large, the government has a stronger incentive to increase public non-tertiary education expenditures to offset the declining marginal utility of pensions driven by higher savings, leading to an increase in the public non-tertiary education expenditure-to-GDP ratio. In contrast, when the relative weight is small, the negative income effect of increased private tertiary education expenditures dominates, prompting a cut in public non-tertiary education expenditures and a reduction in the ratio. Consequently, the response of the public non-tertiary education expenditures to a loosening of the borrowing constraint is positive when the relative weight is high, negative when it is low, and exhibits an inverse U-shaped pattern in intermediate cases.

A decline in the population growth rate increases the relative weight of the older adults in the political objective function, thereby amplifying the importance of pension benefits and raising the pension expenditure-to-GDP ratio. With respect to the labor income tax rate, two opposing effects emerge. On one hand, the greater political weight of older adults heightens the perceived tax burden on them, exerting downward pressure on the tax rate. On the other hand, the reduced relative weight of the middle-aged lowers the political cost associated with taxing their consumption, thereby placing upward pressure on the tax rate. On net, the latter effect dominates, and a decline in the population growth rate ultimately leads to an increase in the tax rate. Furthermore, the public non-tertiary education expenditures increase the return on savings of the middle-aged. As the relative weight on these returns diminishes with a lower population growth rate, the public non-tertiary education expenditure-to-GDP ratio also declines.

To explore the effects of partial public funding of tertiary education in economies with borrowing constraints, the model is calibrated to Japan and the United States, two countries with well-developed education loan markets. Despite the introduction of partial public funding, the qualitative effects on tax rates, public non-tertiary education expenditure, and pension expenditures remain consistent with the benchmark case without partial public funding of tertiary education. In particular, relaxing borrowing constraints reduces both the tax rate and the pension expenditure-to-GDP ratio in both countries. In addition, the public non-tertiary education expenditure-to-GDP ratio responds differently: it increases in Japan and initially rises and then falls in the U.S., reflecting variations in political demographics. Furthermore, as borrowing constraints are relaxed, private tertiary education expenditure increases, which raises the fiscal cost associated with public co-funding and thereby reduces the publicly financed share, resulting in a decline in the public tertiary education expenditure-to-GDP ratio. Nonetheless, the net impact of relaxing borrowing constraints is an increase in total tertiary education expenditures and, importantly, a rise in the steady-state growth rate of GDP per middle-aged individual, driven by improved human capital accumulation despite a mild decline in physical capital accumulation.

A decline in the population growth rate exerts several important effects on fiscal policy and economic growth. The tax rate and pension expenditure-to-GDP ratio both rise, while the public non-tertiary education expenditure-to-GDP ratio declines, consistent with earlier comparative statics results. These changes influence the government's optimal choice of the public funding portion of tertiary education expenditures. Specifically, a lower population growth rate raises the tax rate and reduces disposable income, thereby decreasing private borrowing for tertiary education. The resulting decline in private education expenditure reduces the fiscal cost of public co-funding, prompting the government to raise the public funding portion. However, even as the public funding portion rises, the overall level of tertiary education expenditures falls, resulting in an ambiguous effect on the public tertiary education expenditure-to-GDP ratio. In Japan, this ratio increases monotonically with falling population growth, while in the U.S., it exhibits a hump-shaped pattern. Importantly, the decline in population growth promotes both physical and human capital accumulation—leading to an overall increase in the GDP growth rate per middle-aged individual—because the positive effects of increased savings and relaxed borrowing constraints outweigh the negative effects of rising taxes and pension liabilities.

Using demographic projections from the United Nations World Population Prospects (see Appendix E for the data source), the model tracks the evolution of fiscal variables and growth outcomes through 2100 for Japan and the United States. Japan is projected to experience negative population growth until around 2080, followed by a slight recovery, while the U.S. maintains positive but declining growth. These projections imply that Japan will see rising tax rates and pension expenditures, along with falling public non-tertiary education expenditures until 2080, after which these trends reverse. The public funding portion of tertiary education expenditures is expected to increase until 2080 and decline thereafter. In the U.S., the public funding portion increases more steadily. Despite the increasing public funding portion, the public tertiary education expenditure-to-GDP ratio remains relatively stable in both countries, as the rise in the public share is offset by a decline in overall tertiary education expenditures. Regarding growth, both countries experience declines in labor supply growth due to rising taxes. Japan's growth rate of physical capital per middle-aged individual continues to decline, while human capital growth also slows. The U.S. shows similar trends but with more moderate declines. Overall, economic growth remains positive but decelerates in both nations, with Japan experiencing a sharper downward trajectory.

We conclude the introduction by noting that this paper also contributes to the literature on optimal pension and education policy. Krueger and Ludwig (2016), Andersen and Bhattacharya (2017), Bishnu and Wang (2017), Bishnu et al. (2021, 2023), and Amol et al. (2023) propose pension-education policy packages that achieve Pareto-improving or optimal allocations across periods. While our study shares their concern with intergenerational resource allocation, it differs in its treatment of the government's time horizon. Their analyses rely on the implicit assumption of an infinitely lived government capable of calculating and committing to intergenerationally optimal policies. In contrast, our study assumes that intergenerational resource redistribution must be implemented by a short-lived government that represents only the overlapping generations alive in the current period. Accordingly, resource reallocation is constrained to the current period and cannot rely on long-term commitment.

Section 2 presents the model, including individuals' education, labor, and savings choices, firms' profit maximization, the government budget constraint, and asset market clearing. Section 3 analyzes voting over fiscal policy, derives the resulting fiscal policy, and conducts comparative statics with respect to borrowing constraints and population growth. Section 4 presents a quantitative analysis by calibrating the model to Japan and the United States. Section 5 concludes.

2 Model

The discrete time economy starts in period 0 and consists of overlapping generations. Individuals are identical within a generation and live through three life stages: youth, middle age, and older adult age. Each middle-aged individual has $1+n$ children. The middle-aged population for period t is N_t and the population grows at a constant rate of $n(>-1)$: $N_{t+1} = (1+n)N_t$.

2.1 Individuals

Consider an individual born in period $t-1$. In youth, the individual receives publicly financed non-tertiary education (i.e., primary and secondary education) and subsequently pursues tertiary education. The individual invests e_{t-1} units in tertiary education, of which a fraction σ_{t-1} is publicly financed. Thus, out of the total investment e_{t-1}, the share σ_{t-1} is covered by the government, while the remaining share $1-\sigma_{t-1}$ is financed privately via borrowing in the asset market (described below). Put differently, once the individual chooses the level of tertiary education investment e_{t-1}, the government subsidizes a fraction σ_{t-1} of that amount.

The budget constraints in middle and older adult ages are expressed as:

$$c_t + s_t + R_t\left(1-\sigma_{t-1}\right)e_{t-1} \leq \left(1-\tau_t\right)w_t h_t l_t, \tag{1}$$

$$d_{t+1} \leq R_{t+1}s_t + b_{t+1}. \tag{2}$$

In middle age, the individual is endowed with one unit of time for labor and h_t units of human capital. They supply $l_t \in [0,1]$ units of labor to firms and earn an effective labor income of $w_t h_t l_t$, where w_t denotes the wage rate per unit of effective labor. The individual pays labor income tax in the amount $\tau_t w_t h_t l_t$ where τ_t is the labor income tax rate. The after-tax income is allocated to consumption c_t, savings s_t, and repayment of the privately financed portion of the tertiary education investment undertaken in youth, $R_t\left(1-\sigma_{t-1}\right)e_{t-1}$, where R_t is the gross interest rate between periods $t-1$ and t. In older adult age, the individual receives a gross return on savings, $R_{t+1}s_t$, as well as a pay-as-you-go public pension benefit, b_{t+1}, both of which are devoted to consumption d_{t+1}.

The preferences of the middle-aged in period t are represented by the following lifetime utility function:

$$\ln\left(c_t - \frac{w_t h_t}{\psi}\cdot\frac{(l_t)^{1+1/v}}{1+1/v}\right) + \beta\ln d_{t+1}, \tag{3}$$

where $\beta \in (0,1)$ denotes the subjective discount factor. The disutility of labor supply is specified as $\frac{(l_t)^{1+1/v}}{1+1/v}$, where $v > 0$ parameterizes the Frisch elasticity of labor supply. This formulation is known as the Greenwood–Hercowitz–Huffman (GHH) preference specification (Greenwood et al., 1988). In addition, the disutility of labor is multiplied by the term $w_t h_t / \psi$ where $\psi (> 0)$ is a parameter representing an inverse weight on labor supply disutility. In the present endogenous-growth framework, omitting this term would imply a sustained increase in hours worked over time, eventually violating the time constraint. Including the term ensures the existence of a balanced-growth path with constant labor supply (Queralto, 2020).

In youth, individuals may borrow in the asset market to finance their investment in tertiary education. However, because they have no collateral at the time of borrowing, their borrowing capacity is assumed to be subject to the following constraint:

$$(1 - \sigma_{t-1})\, e_{t-1} \leq \phi \frac{(1 - \tau_t)\, w_t h_t l_t}{R_t}, \tag{4}$$

where the parameter $\phi \in [0, 1]$ represents the degree of borrowing flexibility in the market. Each individual is allowed to borrow up to a fraction ϕ of the present value of their after-tax labor income in middle age. A lower value of ϕ indicates a tighter borrowing constraint. The borrowing limit is determined not by the present value of lifetime after-tax income, but solely by after-tax income earned in middle age. This follows the assumption that future pension benefits cannot be used as collateral for loan repayment, which is standard in the literature (Andersen et al., 2021, 2024).

Let h_t denote the human capital of a period-t representative middle-aged individual. We assume that human capital is determined by the stock inherited from parents and expenditures on non-tertiary and tertiary education. Specifically, human capital evolves according to the following production function:

$$h_t = D\,(h_{t-1})^{1-\eta-\mu}\,(x_{t-1})^{\eta}\,(e_{t-1})^{\mu}\,. \tag{5}$$

Here, h_{t-1} is the human capital inherited by the period-t middle-aged individual from their parents; x_{t-1} is the expenditure on non-tertiary education per young individual in period $t-1$; and e_{t-1} is the tertiary education expenditure per young individual in period $t-1$. The parameters D, η, and μ with $D > 0$, $\eta \in (0, 1)$, $\mu \in (0, 1)$, and $\eta + \mu \in (0, 1)$ are the productivity of human capital production, elasticity of the non-tertiary education expenditure, and elasticity of the tertiary education expenditure, respectively.

The human capital production function has two distinguishing features. First, parental human capital exerts an external effect on the formation of children's human capital—that is, an intergenerational transmission of human capital. We include the previous generation's human capital stock in the human capital production function because empirical evidence shows that a child's educational achievement is positively influenced by that of their parents (see, e.g., Solon, 1999; Black et al., 2005). Since this transmission is not perfect, we allow human capital to depreciate at the rate $1 - \eta - \mu$. Second, non-tertiary education and tertiary education are modeled as imperfect substitutes in the production of human capital: non-tertiary schooling primarily imparts general skills, whereas tertiary education provides specialized skills. The combination of general skills acquired early and specialized training later drives overall human capital formation. In reality, some individuals progress to tertiary education while others do not; however, because the model assumes identical individuals, we interpret the representative individual's tertiary education expenditure as the economy's average private expenditure on tertiary education.

We assume the following regarding the funding of education expenditures. Expenditures on non-tertiary education are assumed to be fully publicly financed, reflecting the fact that in the target countries of this study (Japan and the United States), a substantial share of these expenditures is financed by local and central governments (see Figure 1(a)). In contrast, we assume that expenditures on tertiary education are financed through a mix of public and private sources, reflecting the pattern observed in Figure 1(b) and consistent with empirical evidence from these countries (see Soares, 2005; Diris and Ooghe, 2018). Letting σ denote the policy variable, the shares of public and private funding in tertiary education expenditures are represented by σ and $1-\sigma$, respectively.

The decision-making process of an individual born in period $t-1$ proceeds as follows. In youth, taking the level of public non-tertiary education expenditure x_{t-1} and the portion of public finance for tertiary education σ_{t-1} as given, the individual chooses the level of tertiary education expenditure e_{t-1}. In middle age, given the previously chosen e_{t-1}, the individual determines labor supply l_t and savings s_t. The corresponding levels of consumption, c_t in middle age and d_{t+1} in old age, are determined so as to satisfy the budget constraints. This two-stage optimization problem is solved by backward induction.

2.1.1 Problem of the Middle-aged

The problem of the period-t middle-aged is to choose $\{s_t, l_t\}$ to maximize the lifetime utility in (3) subject to the budget constraints in middle age and older adult age, (1) and (2), respectively, given the predetermined variables, $\{h_t, e_{t-1}\}$, the factor prices, $\{w_t, R_t, R_{t+1}\}$, and the set of policy variables, $\{\tau_t, \sigma_{t-1}, b_{t+1}\}$. Solving the problem leads to the following labor supply and saving functions:

$$l_t = l(\tau_t) \equiv [\psi(1-\tau_t)]^v . \tag{6}$$

$$s_t = \frac{\beta}{1+\beta}\left\{\frac{1/v}{1+1/v}(1-\tau_t)\, l(\tau_t)\, w_t h_t - R_t(1-\sigma_{t-1})\, e_{t-1} - \frac{b_{t+1}}{\beta R_{t+1}}\right\}. \tag{7}$$

Equation (6) indicates that the borrowing constraint does not directly affect labor supply. However, such constraints may influence labor supply decisions, as individuals may substitute leisure for work to mitigate the negative impact of limited consumption (see Lochner and Monge-Naranjo, 2012: Section 2.3). In the present model, this incentive is not explicitly incorporated; thus, the labor supply in (6) remains independent of ϕ. Instead, we focus on the indirect effects of these constraints on labor supply through the tax rate. Equation (7) indicates that savings depend on middle-aged after-tax income net of loan repayments from youth. Furthermore, since older-adult-age pension benefits serve as a substitute for savings, they tend to discourage saving behavior.

Using the labor supply and saving functions in (6) and (7), respectively, with the budget constraints in (1) and (2), we can express the consumption functions in middle age and older adult age as:

$$c_t = \frac{1}{1+\beta}\left\{\left(1+\beta-\beta\frac{1/v}{1+1/v}\right)(1-\tau_t)\,l(\tau_t)\,w_t h_t - R_t(1-\sigma_{t-1})\,e_{t-1} + \frac{b_{t+1}}{R_{t+1}}\right\}, \quad (8)$$

$$d_{t+1} = \frac{\beta}{1+\beta}R_{t+1}\left\{\frac{1/v}{1+1/v}(1-\tau_t)\,l(\tau_t)\,w_t h_t - R_t(1-\sigma_{t-1})\,e_{t-1} + \frac{b_{t+1}}{R_{t+1}}\right\}, \quad (9)$$

and the consumption in middle age net labor supply disutility as:

$$c_t - \frac{w_t h_t}{\psi}\cdot\frac{(l_t)^{1+1/v}}{1+1/v} = \frac{1}{1+\beta}\left\{\frac{1/v}{1+1/v}(1-\tau_t)\,l(\tau_t)\,w_t h_t - R_t(1-\sigma_{t-1})\,e_{t-1} + \frac{b_{t+1}}{R_{t+1}}\right\}. \quad (10)$$

2.1.2 Problem of the Young

We next consider the utility maximization problem of the period-$t-1$ young to derive the optimal level of tertiary education expenditure e_{t-1}. The problem is to choose e_{t-1} that maximizes the sum of the terms in the lifetime utility function that contain e_{t-1}, $(1-\tau_t)\,w_t h_t l(\tau_t) - R_t(1-\sigma_{t-1})\,e_{t-1} - \frac{w_t h_t}{\psi}\cdot\frac{(l(\tau_t))^{1+1/v}}{1+1/v}$, subject to the borrowing constraint in (4) and the human capital production function in (5), given the labor supply in (6). The first-order condition yields the following solution:

$$(1-\sigma_{t-1})\,e_{t-1} = \begin{cases} \frac{\mu}{R_t}\left[(1-\tau_t)\,w_t h_t l(\tau_t) - \frac{w_t h_t}{\psi}\cdot\frac{(l(\tau_t))^{1+1/v}}{1+1/v}\right] & \text{if } \mu\frac{1/v}{1+1/v}\le\phi, \\ \phi\frac{(1-\tau_t)w_t h_t l(\tau_t)}{R_t} & \text{otherwise.}\end{cases}$$

Assumption 1: $\mu\frac{1/v}{1+1/v} > \phi$.

Since our objective is to examine how the degree of borrowing constraints affects policy outcomes, in the following we impose Assumption 1 and focus on the case in which the young face a binding borrowing constraint. In this case, private tertiary education expenditure is given by:

$$(1-\sigma_{t-1})\,e_{t-1} = \phi\frac{(1-\tau_t)\,w_t h_t l(\tau_t)}{R_t}. \quad (11)$$

2.2 Firms

There is a continuum of identical firms that are perfectly competitive profit maximizers and produce the final output Y_t with a constant-returns-to-scale Cobb-Douglas production function, $Y_t = A(K_t)^{\alpha}(L_t)^{1-\alpha}$, where $A(>0)$ denotes total factor productivity, which is assumed to be constant over time; K_t is aggregate physical capital, $L_t \equiv H_t l_t$ is aggregate effective labor, $H_t \equiv h_t N_t$ is aggregate human capital, and $\alpha \in (0,1)$ is a constant parameter representing the share of physical capital in production.

In each period, firms choose physical capital and effective labor to maximize its profit, $A(K_t)^{\alpha}(L_t)^{1-\alpha} - R_t K_t - w_t L_t$, where R_t is the gross return on physical capital and w_t is the real wage per unit of effective labor. Profit maximization yields the following first-order conditions:

$$K_t : R_t = \alpha A (K_t)^{\alpha-1} (L_t)^{1-\alpha} = \alpha A (k_t)^{\alpha-1} (h_t)^{1-\alpha} (l_t)^{1-\alpha}, \tag{12}$$

$$L_t : w_t = (1-\alpha) A (K_t)^{\alpha} (L_t)^{-\alpha} = (1-\alpha) A (k_t)^{\alpha} (h_t)^{-\alpha} (l_t)^{-\alpha}, \tag{13}$$

where $k_t \equiv K_t/N_t$ and $h_t \equiv H_t/N_t$ denote physical and human capital per middle-aged individual, respectively.

Let $y_t \equiv Y_t/N_t$ denote output per middle-aged individual. The corresponding per-middle-aged individual production function is $y_t = A(k_t)^{\alpha}(h_t)^{1-\alpha}(l_t)^{1-\alpha}$, which can be rewritten as:

$$y_t = y(k_t, h_t, \tau_t) \equiv \hat{y}(k_t, h_t)\, z(\tau_t), \tag{14}$$

where $\hat{y}(k_t, h_t)$ and $z(\tau_t)$ are defined by:

$$\hat{y}(k_t, h_t) \equiv A (k_t)^{\alpha} (h_t)^{1-\alpha}, \tag{15}$$

$$z(\tau_t) \equiv [\psi(1-\tau_t)]^{v(1-\alpha)}. \tag{16}$$

The term $\hat{y}(k_t, h_t)$ represents the output per middle-aged individual under inelastic labor supply, and the term $z(\tau)$ captures the contribution of labor supply to production. When $v = 0$, labor supply is inelastic and tax distortions disappear, implying $z(\cdot) = 1$.

2.3 Government Budget

The government expenditures inlcude investment in non-tertiary education, subsidies for private investment in tertiary education, and pension payments. They are financed by taxes on labor income. The government budget constraint in period t is $N_{t+1}(x_t + \sigma_t e_t) + N_{t-1} b_t = N_t \tau_t w_t h_t l_t$, where $N_{t+1} x_t$ is the aggregate investment on non-tertiary education, $N_{t+1}\sigma_t e_t$ is the aggregate subsidy expenditure for private tertiary education, $N_{t-1} b_t$ is the aggregate pension payment, and $N_t \tau_t w_t h_t l_t$ is the aggregate labor income tax revenue. The constraint on a per middle-aged individual basis is:

$$(1+n)(x_t + \sigma_t e_t) + \frac{b_t}{1+n} = \tau_t w(k_t, h_t, \tau_t) h_t l(\tau_t). \tag{17}$$

2.4 Economic Equilibrium

An asset market operates in this model economy, facilitating borrowing and lending activities. In this market, the middle-aged supply funds $N_t s_t$, which are borrowed by firms for investment in physical capital, K_{t+1}, and by the young for investment in human capital, $N_{t+1}(1-\sigma_t) e_t$. The asset market clearing condition is $K_{t+1} + N_{t+1}(1-\sigma_t) e_t = N_t s_t$, which is reformulated as follows:

$$(1+n)(k_{t+1} + (1-\sigma_t) e_t) = s_t. \tag{18}$$

The timing of events in period t is as follows. Initially, the voting stage takes place given the state variables k_t and h_t with the predetermined private tertiary education expenditures, $(1-\sigma_{t-1})\,e_{t-1}$. The government determines a set of fiscal policies, τ_t, x_t, b_t, and σ_t (described in Section 3). Subsequently, the economic decision stage follows given the fiscal policies. Individuals and firms make their decisions, leading to market clearing. Thus, an economic equilibrium, characterizing optimizing behavior of individuals and firms and the resulting market clearing for a given set of fiscal policies, is defined as follows:

Definition 1 *Given a sequence of policies, $\{\tau_t, x_t, b_t, \sigma_t\}_{t=0}^{\infty}$, an economic equilibrium is a sequence of allocations $\{c_t, d_t, s_t, e_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ and prices $\{w_t, R_t\}_{t=0}^{\infty}$ with initial conditions $k_0(>0)$ and $h_0(>0)$ and the pre-determined private tertiary education expenditures $(1-\sigma_{-1})\,e_{-1}$, such that (i) given $(h_t, w_t, R_t, R_{t+1}, \tau_t, b_{t+1})$ and $(1-\sigma_{t-1})\,e_{t-1}$, (c_t, d_{t+1}, s_t) solves the utility-maximizing problem of the middle-aged; and given $(h_t, w_{t+1}, R_{t+1}, x_t, \sigma_t, \tau_{t+1})$, e_t solves the utility maximization problem of the young; (ii) given (w_t, R_t), k_t solves the firm's profit maximization problem; (iii) given (h_t, w_t, e_t, l_t), $(\tau_t, x_t, b_t, \sigma_t)$ satisfies the government budget constraint; and (iv) the asset market clears.*

Given the borrowing constraint faced during youth, the individual's utility maximization yields the labor supply in (6), saving in (7), consumption in (8) and (9), and tertiary education expenditure in (11). The firm's profit maximization yields the physical capital demand in (12) and labor demand in (13). Combining the labor supply in (6) and the labor demand in (13), the equilibrium wage that clears the labor market is given by:

$$w_t = w\left(k_t, h_t, \tau_t\right) \equiv (1-\alpha)\,\hat{y}\left(k_t, h_t\right) z\left(\tau_t\right)^{-\alpha/(1-\alpha)} / h_t, \tag{19}$$

Based on the labor supply function in (6) and the physical capital demand function in (12), the gross interest rate (representing the rental price of physical capital) is determined as follows:

$$R_t = R\left(k_t, h_t, \tau_t\right) \equiv \alpha \hat{y}\left(k_t, h_t\right) z\left(\tau_t\right) / k_t. \tag{20}$$

Using (19) and (20), we write the saving in (7) and tertiary education expenditure in (11) as functions of the state and policy variables as follows:

$$\begin{aligned} s_t &= \bar{s}(\tau_t, k_t, h_t, (1-\sigma_{t-1})\,e_{t-1}, \tau_{t+1}, b_{t+1}, k_{t+1}, h_{t+1}) \\ &\equiv \frac{\beta}{1+\beta}\left\{ I\left(\tau_t, k_t, (1-\sigma_{t-1})\,e_{t-1}\right) \hat{y}\left(k_t, h_t\right) z\left(\tau_t\right) - \frac{b_{t+1}}{\beta R\left(k_{t+1}, h_{t+1}, \tau_{t+1}\right)} \right\}, \end{aligned} \tag{21}$$

$$e_t = \bar{e}(\sigma_t, \tau_{t+1}, k_{t+1}) \equiv \frac{1}{1-\sigma_t} \phi\left(1-\tau_{t+1}\right) \frac{1-\alpha}{\alpha} k_{t+1}, \tag{22}$$

where $I\left(\cdot\right)$, representing the after-tax income of the middle-aged net of labor costs and youth loan repayment, evaluated per unit of output, is defined as follows:

$$I\left(\tau_t, k_t, (1-\sigma_{t-1})\,e_{t-1}\right) \equiv \frac{1/v}{1+1/v}\left(1-\tau_t\right)(1-\alpha) - (1-\sigma_{t-1})\,e_{t-1}\frac{\alpha}{k_t}. \tag{23}$$

Next, we derive the physical and human capital accumulation equations in the economic equilibrium. We first rewrite the asset market clearing condition (18). Combining (18) with the saving function (21) and the tertiary education expenditure function (22) yields:

$$\begin{aligned}&(1+n)(k_{t+1}+(1-\sigma_t)\,\bar{e}(\sigma_t,\tau_{t+1},k_{t+1}))\\&=\bar{s}(\tau_t,k_t,h_t,(1-\sigma_{t-1})\,e_{t-1},\tau_{t+1},b_{t+1},k_{t+1},h_{t+1}).\end{aligned}\tag{24}$$

Using the human capital production function in (5) together with (22), we obtain:

$$h_{t+1}=D\,(h_t)^{1-\eta-\mu}\,(x_t)^{\eta}\,(\bar{e}(\sigma_t,\tau_{t+1},k_{t+1}))^{\mu}\,.\tag{25}$$

Substituting (25) into (24) and solving implicitly for k_{t+1} yields:

$$k_{t+1}=\bar{k}'\left(\tau_t,\sigma_t,x_t,k_t,h_t,(1-\sigma_{t-1})\,e_{t-1},\tau_{t+1},b_{t+1}\right).\tag{26}$$

Because $(1-\sigma_{t-1})\,e_{t-1}$ is predetermined in period $t-1$, it is treated as a predetermined variable when solving for equilibrium in period t.

Substituting (26) into (25) gives the corresponding human capital accumulation equation:

$$\begin{aligned}h_{t+1}&=\bar{h}'\left(\tau_t,\sigma_t,x_t,k_t,h_t,(1-\sigma_{t-1})\,e_{t-1},\tau_{t+1},b_{t+1}\right)\\&\equiv D\,(h_t)^{1-\eta-\mu}\,(x_t)^{\eta}\left(\bar{e}(\sigma_t,\tau_{t+1},\bar{k}'\left(\tau_t,\sigma_t,x_t,k_t,h_t,(1-\sigma_{t-1})\,e_{t-1},\tau_{t+1},b_{t+1}\right))\right)^{\mu}.\end{aligned}\tag{27}$$

Equations (26) and (27) therefore describe the law of motion for next-period physical and human capital, respectively, conditional on current-period physical and human capital, the predetermined variable $(1-\sigma_{t-1})\,e_{t-1}$, and the set of fiscal policy variables.

Using the physical and human capital accumulation equations in (26) and (27), the saving function (21) and the tertiary education expenditure function (22) can be rewritten as:

$$\begin{aligned}s_t&=s\left(\tau_t,\sigma_t,x_t,k_t,h_t,(1-\sigma_{t-1})\,e_{t-1},\tau_{t+1},b_{t+1}\right)\\&\equiv\frac{\beta}{1+\beta}\left\{I\left(\tau_t,k_t,(1-\sigma_{t-1})\,e_{t-1}\right)\hat{y}\left(k_t,h_t\right)z\left(\tau_t\right)-\frac{b_{t+1}}{\beta R\left(\bar{k}'\left(\cdot\right),\bar{h}'\left(\cdot\right),\tau_{t+1}\right)}\right\},\end{aligned}\tag{28}$$

$$\begin{aligned}e_t&=e\left(\tau_t,\sigma_t,x_t,k_t,h_t,(1-\sigma_{t-1})\,e_{t-1},\tau_{t+1},b_{t+1}\right)\\&\equiv\frac{1}{1-\sigma_t}\phi\left(1-\tau_{t+1}\right)\frac{1-\alpha}{\alpha}\bar{k}'\left(\tau_t,\sigma_t,x_t,k_t,h_t,(1-\sigma_{t-1})\,e_{t-1},\tau_{t+1},b_{t+1}\right).\end{aligned}\tag{29}$$

Accordingly, savings s_t and tertiary education expenditure e_t in period t can be expressed as functions of the policy variables $\{\tau_t,\sigma_t,x_t,\tau_{t+1},b_{t+1}\}$, the current state variables $\{k_t,h_t\}$, and the predetermined variable $(1-\sigma_{t-1})\,e_{t-1}$. The government budget constraint (17) can also be rewritten, using (6), (19), and (29), as:

$$(1+n)\left(x_t+\sigma_t e\left(\cdot\right)\right)+\frac{b_t}{1+n}=\tau_t\left(1-\alpha\right)\hat{y}\left(k_t,h_t\right)z\left(\tau_t\right).\tag{30}$$

Finally, we derive the consumption functions in the economic equilibrium and, based on them, the corresponding indirect utility functions of the middle-aged and older adults. We begin with consumption in middle age, $c(\cdot)$. From (8), together with the factor prices in (19) and (20) and the physical and human capital accumulation equations in (26) and (27), we obtain:

$$\begin{aligned} c_t &= c\left(\tau_t, \sigma_t, x_t, k_t, h_t, (1-\sigma_{t-1}) e_{t-1}, \tau_{t+1}, b_{t+1}\right) \\ &\equiv \frac{1}{1+\beta}\left\{\left(1+\beta-\beta\frac{1/v}{1+1/v}\right)(1-\tau_t)(1-\alpha)\hat{y}(k_t,h_t) z(\tau_t)\right. \\ &\left. -(1-\sigma_{t-1}) e_{t-1}\hat{y}(k_t,h_t) z(\tau_t)\frac{\alpha}{k_t} + \frac{b_{t+1}}{R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau_{t+1}\right)}\right\}. \end{aligned} \tag{31}$$

Using (19), (20), (26), and (27) together with the middle-aged consumption in (31), we can reformulate the consumption in middle age net labor disutility, $c_t - \frac{w_t h_t}{\psi} \cdot \frac{(l_t)^{1+1/v}}{1+1/v}$ in (10), as follows:

$$\begin{aligned} & c_t - \frac{w_t h_t}{\psi} \cdot \frac{(l_t)^{1+1/v}}{1+1/v} \\ &= \frac{1}{1+\beta}\left\{I\left(\tau_t, k_t, (1-\sigma_{t-1}) e_{t-1}\right)\hat{y}(k_t,h_t) z(\tau_t) + \frac{b_{t+1}}{R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau_{t+1}\right)}\right\}. \end{aligned} \tag{32}$$

Consumption in older adult age, $d'(\cdot)$, in (9), is reformulated, using (19), (20), (26), and (27) as follows:

$$\begin{aligned} d_{t+1} &= d'\left(\tau_t, \sigma_t, x_t, k_t, h_t, (1-\sigma_{t-1}) e_{t-1}, \tau_{t+1}, b_{t+1}\right) \\ &\equiv \frac{\beta}{1+\beta} R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau_{t+1}\right) \\ &\times \left\{I\left(\tau_t, k_t, (1-\sigma_{t-1}) e_{t-1}\right)\hat{y}(k_t,h_t) z(\tau_t) + \frac{b_{t+1}}{R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau_{t+1}\right)}\right\}. \end{aligned} \tag{33}$$

With (32) and (33), we can write the indirect utility of the middle-aged, V_t^M, as follows:

$$\begin{aligned} V_t^M &= V^M\left(\tau_t, \sigma_t, x_t, k_t, h_t, (1-\sigma_{t-1}) e_{t-1}, \tau_{t+1}, b_{t+1}\right) \\ &\equiv (1+\beta)\ln\left\{I\left(\tau_t, k_t, (1-\sigma_{t-1}) e_{t-1}\right)\hat{y}(k_t,h_t) z(\tau_t) + \frac{b_{t+1}}{R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau_{t+1}\right)}\right\} \\ &+ \beta\ln R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau_{t+1}\right) + \left(\ln\frac{1}{1+\beta} + \beta\ln\frac{\beta}{1+\beta}\right), \end{aligned} \tag{34}$$

The indirect utility of the older adults is given by $V_t^O = \ln(R_t s_{t-1} + b_t) = \ln(1+n)\left(\frac{1}{1+n}R_t s_{t-1} + \frac{1}{1+n}b_t\right)$. Using the asset market clearing condition from one period earlier in (18), we can rewrite V_t^O as

$$V_t^O = V^O\left(\tau_t, b_t, k_t, h_t, (1-\sigma_{t-1}) e_{t-1}\right) \equiv \ln d\left(\tau_t, b_t, k_t, h_t, (1-\sigma_{t-1}) e_{t-1}\right), \tag{35}$$

where

$$d\left(\tau_t, b_t, k_t, h_t, (1-\sigma_{t-1}) e_{t-1}\right) \equiv (1+n)\left[R(k_t, h_t, \tau_t)\left(k_t + (1-\sigma_{t-1}) e_{t-1}\right) + \frac{b_t}{1+n}\right]. \tag{36}$$

3 Political Equilibrium

This section considers voting on fiscal policies. In each period, the young, middle-aged, and older adults have an incentive to vote. While the young may benefit from current public expenditure on tertiary education, we assume that they do not participate in voting. We impose this assumption, which is often used in the literature (Saint-Paul and Verdier, 1993; Bernasconi and Profeta, 2012; Lancia and Russo, 2016; Bishnu and Wang, 2017) for tractability. However, the assumption could be partly justified by the fact that a large number of the young are ineligible to vote due to age restrictions.

We employ probabilistic voting in the spirit of (Lindbeck and Weibull, 1987; Acemoglu and Robinson, 2005), in which there is electoral competition between two political parties. In each period, each party announces a set of fiscal policies to maximize its expected vote share given the policy announcement by the other party. The two parties' platforms converge in equilibrium to the same set of fiscal policies that maximize the political objective, namely, the weighted sum of utility of voters (Persson and Tabellini, 2002; Acemoglu and Robinson, 2005). The objective is given by $\omega V_t^O + (1+n)(1-\omega) V_t^M$, where $\omega \in (0,1)$ and $1-\omega$ are the political weights placed on the older adults and middle-aged, respectively. The weight on the middle-aged is adjusted by the gross population growth rate, $1+n$, to reflect their share of the population. Dividing the objective function by $(1+n)(1-\omega)$ yields the following expression, denoted by Ω_t:

$$\Omega_t = \frac{\omega}{(1+n)(1-\omega)} V_t^O + V_t^M,$$

where the coefficient $\omega/(1+n)(1-\omega)$ represents the relative political weight assigned to the older adults.

We employ the Markov-perfect political equilibrium concept, in which today's fiscal policy depends on the current payoff-relevant state variables.[1] In the current framework, the payoff-relevant state variables in period t are the physical capital, k_t, and the human capital, h_t. Consequently, the expected tax and provision of public pension in period $t+1$, τ_{t+1} and b_{t+1}, could be given by the functions of the period $t+1$ payoff-relevant state variables, k_{t+1} and h_{t+1}: $\tau_{t+1} = T(k_{t+1}, h_{t+1})$ and $b_{t+1} = B(k_{t+1}, h_{t+1})$.

Using the notation, with z_{-1} and z' denoting the pervious period and the next period of z,

[1] The concept enables us to demonstrate voters' forward-looking decision-making and the resulting feedback mechanism between current and future redistributive policies, mediated through physical and human capital accumulation. Studies that account for this mechanism include, for example, Beauchemin (1998), Forni (2005), Bassetto (2008), Mateos-Planas (2008), Gonzalez-Eiras and Niepelt (2012), Song (2011), Song et al. (2012), Chen and Song (2014), Arai et al. (2018), Arcalean (2018), and Uchida and Ono (2021, 2024a,b).

respectively, we can write the government problem as follows:

$$\begin{aligned}
&\max_{\{\tau,x,b,\sigma\}} \Omega\left(\tau,\sigma,x,b,k,h,(1-\sigma_{-1})e_{-1},\tau',b'\right)\\
&\equiv \frac{\omega}{(1+n)(1-\omega)}V^{O}\left(\tau,b,k,h,(1-\sigma_{-1})e_{-1}\right)+V^{M}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\tau',b'\right)\\
&\text{s.t. } (1+n)\left[x+\sigma e\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\tau',b'\right)\right]+\frac{b}{1+n}=\tau(1-\alpha)\hat{y}(k,h)z(\tau), \quad (37)\\
&k'=\bar{k}'\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\tau',b'\right), \quad (38)\\
&h'=\bar{h}'\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\tau',b'\right), \quad (39)
\end{aligned}$$

with $\tau'=T(k',h')$ and $b'=B(k',h')$, where (37) is the government budget constraint from (30), (38) is the physical capital accumulation equation from (26), and (39) is the human capital accumulation equation from (27). A Markov-perfect political equilibrium in the present framework is defined as follows:

Definition 2 *A Markov-perfect political equilibrium is a six-tuple, $(\hat{T},\hat{\Sigma},\hat{X},\hat{B},\hat{S},\hat{E})$, where $\hat{T}:\Re_{+}^{2}\to[0,1]$ is the tax rule, $\tau=\hat{T}(k,h)$; $\hat{\Sigma}:\Re_{+}^{2}\to[0,1]$ is the public funding rule, $\sigma=\hat{\Sigma}(k,h)$; $\hat{X}:\Re_{+}^{2}\to\Re_{+}$ is the public non-tertiary education expenditure rule, $x=\hat{X}(k,h)$; $\hat{B}:\Re_{+}^{2}\to\Re_{+}$ is the pension expenditure rule, $b=\hat{B}(k,h)$; $\hat{S}:[0,1]^{2}\times\Re_{+}^{4}\to\Re_{+}$ is the saving rule, $s=\hat{S}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1}\mid\hat{T},\hat{B}\right)$; and $\hat{E}:[0,1]^{2}\times\Re_{+}^{4}\to\Re_{+}$ is the tertiary education expenditure rule, $e=\hat{E}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1}\mid\hat{T},\hat{B}\right)$, such that (i) for a given $\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\hat{T}$ and $\hat{B}$, the optimal private saving and tertiary education expenditure rules are the maps, $\hat{S}$ and $\hat{E}$, respectively, that solve*

$$\hat{S}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1}\mid\hat{T},\hat{B}\right)=s\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\hat{T}\left(\hat{S}(\cdot),\hat{E}(\cdot)\right),\hat{B}\left(\hat{S}(\cdot),\hat{E}(\cdot)\right)\right),$$
$$\hat{E}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1}\mid\hat{T},\hat{B}\right)=e\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\hat{T}\left(\hat{S}(\cdot),\hat{E}(\cdot)\right),\hat{B}\left(\hat{S}(\cdot),\hat{E}(\cdot)\right)\right),$$

where $\tau'=\hat{T}(k',h')$ and $b'=\hat{B}(k',h')$ with $k'=\bar{k}'\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\hat{T},\hat{B}\right)$, $h'=\bar{h}'\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\hat{T},\hat{B}\right)$ and $(1-\sigma_{-1})e_{-1}=\phi(1-\tau)(1-\alpha)k/\alpha$; (ii) given the set of initial conditions (k,h) with $(1-\sigma_{-1})e_{-1}$ and the political objective function

$$\begin{aligned}
&\Omega\left(\tau,\sigma,x,b,k,h,(1-\sigma_{-1})e_{-1},\tau',b'\right)\\
&=\frac{\pi\omega}{(1+n)(1-\omega)}V^{O}\left(\tau,b,(1-\sigma_{-1})e_{-1},k,h\right)+V^{M}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\tau',b'\right)
\end{aligned}$$

where $\tau'=\hat{T}(k',h')$ and $b'=\hat{B}(k',h')$ with $k'=\bar{k}'\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\hat{T},\hat{B}\right)$ and $h'=\bar{h}'\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})e_{-1},\hat{T},\hat{B}\right)$, (a) the equilibrium fiscal policies solve

$$\left(\hat{T}(k,h),\hat{\Sigma}(k,h),\hat{X}(k,h),\hat{B}(k,h)\right)=\arg\max_{\tau,\sigma,x,b}\Omega\left(\tau,\sigma,x,b,k,h,(1-\sigma_{-1})e_{-1}\mid\hat{T},\hat{B}\right)$$

subject to the government budget constraint $(1+n)(x+\sigma e)+b/(1+n)=\tau(1-\alpha)\hat{y}(k,h)z(\tau)$, (b) the tertiary education expenditure function satisfies

$$(1-\sigma_{-1})e_{-1}=\phi\left(1-\hat{T}(k,h)\right)(1-\alpha)k/\alpha,$$

and (c) the government budget constraint satisfies

$$(1+n)\left(\hat{X}(k,h)+\hat{\Sigma}(k,h)\,\hat{E}\left(\tau,\sigma,x,k,h,\phi\left(1-\hat{T}(k,h)\right)\frac{1-\alpha}{\alpha}k\mid\hat{T},\hat{B}\right)\right)+\frac{\hat{B}(k,h)}{1+n}$$
$$=(1-\alpha)\,\hat{y}(k,h)\,z\left(\hat{T}(k,h)\right).$$

Part (i) defines the functional equations that map current tax, portion of public funding in tertiary education, public non-tertiary education expenditure, physical and human capital stocks, and the borrowing in youth to optimal private saving and tertiary education expenditures, $s=\hat{S}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})\,e_{-1}\mid\hat{T},\hat{B}\right)$ and $e=\hat{E}\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})\,e_{-1}\mid\hat{T},\hat{B}\right)$. This set of rules describes the private sectors' response to changes in τ, σ, and x under the expectation that future tax and pensions will be set according to the equilibrium rules $\hat{T}(k,h)$ and $\hat{B}(k,h)$, respectively. Part (ii) describes the government's problem. In each period, the government sets fiscal policies subject to its budget constraints and the private sectors' response, consistent with the expectation that future governments follow the Markov-perfect political equilibrium rules. Treating $(1-\sigma_{-1})\,e_{-1}$ as given, we first solve the government's problem. We then substitute $(1-\sigma_{-1})\,e_{-1}=\phi(1-\tau)(1-\alpha)\,k/\alpha$ into the solution, thereby expressing $\hat{T},\hat{\Sigma},\hat{X}$ and $\hat{B}$ as functions of k and h.

3.1 Characterization of Political Equilibrium

To obtain the set of policy functions, we conjecture the following policy functions of tax and pension benefits in the next period:

$$\tau'=\bar{T}, \tag{40}$$

$$\frac{\frac{b'}{1+n}}{\hat{y}(k',h')\,z(\tau')}=\bar{B}, \tag{41}$$

where $\bar{T}\,(>0)$ and $\bar{B}\,(>0)$ are constant. Given these conjectures, we can reformulate (38) and (39) as follows (see Appendix A for the derivation)：

$$k'=\tilde{k}'\left(\tau,k,h,(1-\sigma_{-1})\,e_{-1},\bar{T},\bar{B}\right)\equiv\frac{1}{1+n}\frac{\frac{\beta}{1+\beta}I\left(\tau,k,(1-\sigma_{-1})\,e_{-1}\right)\hat{y}(k,h)\,z(\tau)}{1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}, \tag{42}$$

$$h'=\tilde{h}'\left(\tau,\sigma,x,k,h,(1-\sigma_{-1})\,e_{-1},\bar{T},\bar{B}\right)\equiv D\,(h)^{1-\eta-\mu}\,(x)^{\eta}\left(\frac{1}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\tilde{k}'(\cdot)\right)^{\mu}, \tag{43}$$

where k' and h' are related to the net after-tax income, $I(\cdot)\,\hat{y}(k,h)\,z(\tau)$.

Using the indirect utility functions of the middle-aged and older adults in (34) and (35), respectively, together with (42) and (43), the political objective function Ω is expressed by

$$\Omega\approx\frac{\omega}{(1+n)(1-\omega)}\ln d\left(\tau,\frac{b}{1+n},k,h,(1-\sigma_{-1})\,e_{-1}\right)$$
$$+(1+\beta)\ln\left\{I\left(\tau,k,(1-\sigma_{-1})\,e_{-1}\right)\hat{y}(k,h)\,z(\tau)+\frac{1+n}{\alpha}\bar{B}\tilde{k}'(\cdot)\right\}+\beta\ln R\left(\tilde{k}'(\cdot),\tilde{h}'(\cdot),\bar{T}\right), \tag{44}$$

where the first term is derived from V^O; the second and third terms are jointly derived from V^M; and the politically irrelevant terms are omitted from the expression. Under the conjecture in (41), we have $b'/R' = (1+n)\bar{B}\tilde{k}'(\cdot)/\alpha$; this appears in the second term of (44), implying that the present value of the pension benefit, b'/R', is determined solely by the current labor income tax rate, τ.

Below, we consider the choice of $(1+n)x$ and $b/(1+n)$ rather than x and b for the convenience of the following analysis and interpretation of the results. We obtain the corresponding policy functions and verify that the conjectures in (40) and (41) are indeed consistent.

Proposition 1 *The policy functions in the Markov-perfect political equilibrium are expressed by*

$$\tau = \bar{T},\ \sigma = \bar{\Sigma},\ (1+n)\,x/\hat{y}(\cdot)\,z(\cdot) = \bar{X},\ \frac{b}{1+n}/\hat{y}(\cdot)\,z(\cdot) = \bar{B},$$

where $\bar{T}$, $\bar{\Sigma}$, $\bar{X}$, and $\bar{B}$ are constant and depend on parameters only. Given the initial condition, h_0/k_0, the associated sequence of human-to-physical capital ratio, $\{h/k\}$, stably converges to a unique steady state.

Proof. See Appendix A.

To interpret the results established in Proposition 1, we explore the first-order condition with respect to $b/(1+n)$, $(1+n)x$, σ and τ. These conditions are given as follows:

$$\frac{b}{1+n} : \frac{\omega}{(1+n)(1-\omega)} \frac{\partial d(\cdot)/\partial \frac{b}{1+n}}{d(\cdot)} - \lambda = 0, \tag{45}$$

$$(1+n)\,x : \beta \frac{\partial R'/\partial (1+n)\,x}{R'} - \lambda = 0, \tag{46}$$

$$\sigma : \frac{\beta \frac{\partial R'/\partial \sigma}{R'}}{(1+n)\left[e(\cdot) + \sigma \frac{\partial e}{\partial \sigma}\right]} - \lambda = 0, \tag{47}$$

$$\tau : MCT - \lambda = 0, \tag{48}$$

where λ is the Lagrange multiplier associated with the government budget constraint in (37). The marginal cost of taxation (MCT) is defined as follows:

$$MCT \equiv (-1)\frac{\frac{\omega}{(1+n)(1-\omega)}\frac{\partial d(\cdot)/\partial \tau}{d(\cdot)} + (1+\beta)\frac{\frac{\partial I(\cdot)\hat{y}(k,h)z(\tau)}{\partial \tau} + \frac{\partial (b'/R')}{\partial \tau}}{I(\cdot)\hat{y}(k,h)z(\tau) + \frac{b'}{R'}} + \beta \frac{\partial R'/\partial \tau}{R'}}{(1-\alpha)\left(1+\tau\frac{z'(\tau)}{z(\tau)}\right)\hat{y}(k,h)\,z(\tau) + (-1)(1+n)\,\sigma \frac{\partial e}{\partial \tau}}. \tag{49}$$

The first terms on the left-hand sides of (45) and (46) represent the marginal utilities associated with increases in pension benefits $b/(1+n)$ for the older adults and public non-tertiary education expenditures $(1+n)x$ for the young, respectively, both evaluated per unit increase in government spending. In the former case, the marginal utility arises from the direct increase in consumption of the older adults, while in the latter, it results from the rise in next-period human capital, h', which raises the future interest rate, R', and increases the consumption of the middle-aged in their older adult age, d'. Because a one-unit increase in either $b/(1+n)$

or $(1+n)x$ corresponds to a one-unit increase in government expenditure, the second terms in both equations could be expressed as $1 \times \lambda$, where "1" denotes the unit of additional spending. Similarly, the first term in (47) captures the marginal utility of raising the portion of public funding for tertiary education, σ, which increases total education expenditures e, enhances future human capital, and thereby raises the interest rate and future consumption. This marginal utility is normalized by the resulting increase in publicly funded tertiary education expenditures, yielding the marginal utility per unit of public funding. Lastly, the first term in (48) denotes the marginal disutility of taxation per unit increase in tax revenue, reflecting the welfare cost of raising additional funds through taxation.

Finally, combining the government budget constraint in (37) with the physical capital accumulation equation in (42), we obtain

$$(1+n)x + \frac{\sigma}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\frac{\frac{\beta}{1+\beta}}{1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}I\left(\cdot\right)\hat{y}\left(k,h\right)z\left(\tau\right)+\frac{b}{1+n}$$
$$= \tau\left(1-\alpha\right)\hat{y}\left(k,h\right)z\left(\tau\right). \tag{50}$$

Thus, given the conjecture for τ' in (40) and b' in (41), the system consisting of the four optimality conditions, (45) through (48), and the government budget constraint (50) characterizes the policy functions $b/(1+n)$, $(1+n)x$, τ, and σ and the Lagrange multiplier λ.

MCT in (49), representing the marginal disutility of taxation evaluated in terms of changes in the net tax revenue, has the following properties. An increase in the tax rate negatively affects the consumption of both the older adults and middle-aged, as indicated by the numerator in (49). Conversely, the increase in the tax rate leads to a rise in tax revenue and reduces expenditure on tertiary education by alleviating pressure on the government budget, as shown in the denominator of (49). Therefore, the expression in (49) shows the ratio of costs to benefits of taxation, thereby representing the marginal cost of taxation.

To enhance understanding of MCT, we analyze the effects of the labor income tax rate τ on each component of MCT, as defined in (49). First, we focus on the first term of the numerator, $\frac{\omega}{(1+n)(1-\omega)} \cdot \frac{\partial d(\cdot)/\partial\tau}{d(\cdot)}$, which reflects the impact of τ on the consumption of the older adults, $d\left(\cdot\right)$, given by (36). An increase in τ leads to a decrease in the labor supply $l\left(\tau\right)$, thereby decreasing the marginal productivity of physical capital. Consequently, the interest rate R declines. As a result, a rise in τ reduces the return on savings and, hence, the consumption of the older adults: $\partial d\left(\cdot\right)/\partial\tau < 0$. This effect is amplified by the relative political weight of the older adults, $\frac{\omega}{(1+n)(1-\omega)}$.

Next, we turn to the second and third terms of the numerator in (49). They capture the impact of the labor income tax rate τ on the indirect utility of the middle-aged in (34), $V^M\left(\cdot\right)$, expressed by the second and third terms of (44). The tax rate τ thus influences $V^M\left(\cdot\right)$ through three components: $I\left(\tau, k, \left(1-\sigma_{-1}\right)e_{-1}\right)\hat{y}\left(k,h\right)z\left(\tau\right)$, $b'/R' = (1+n)\bar{B}\tilde{k}'\left(\cdot\right)/\alpha$, and $R' = R\left(\tilde{k}'\left(\cdot\right), \tilde{h}'\left(\cdot\right), \bar{T}\right)$. In the following, we analyze how the tax rate affects each of these components.

The term $I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau)$ represents the net after-tax income, which is reformulated as:

$$I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau) = \frac{1/v}{1+1/v} (1-\tau) w(k,h,\tau) hl(\tau) - R(k,h,\tau)(1-\sigma_{-1}) e_{-1}.$$

An increase in τ reduces the labor supply $l(\tau)$, though this reduction is partially offset by an increase in the wage rate $w(k,h,\tau)$. The overall effect on $w(k,h,\tau) hl(\tau)$ is negative. Given the direct negative impact of τ through the term $(1-\tau)$, a rise in τ leads to a decline in after-tax labor income: $\partial (1-\tau) w(k,h,\tau) hl(\tau) / \partial \tau < 0$.

The term $R(k,h,\tau)(1-\sigma_{-1}) e_{-1}$, representing youth loan repayments, decreases with an increase in τ: $\partial R(k,h,\tau)(1-\sigma_{-1}) e_{-1} / \partial \tau < 0$. A rise in τ reduces labor supply $l(\tau)$, which subsequently lowers the marginal productivity of physical capital. As a result, the interest rate R declines, leading to a reduction in youth loan repayments. Thus, an increase in τ exerts two opposing effects on $I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau)$, but the overall impact is negative:

$$\frac{\partial I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau)}{\partial \tau} < 0. \tag{51}$$

The term $b'/R' = (1+n)\bar{B}\tilde{k}'(\cdot)/\alpha$ represents the present value of pension benefits, which is reformulated as follows:

$$\frac{b'}{R'} = \frac{\bar{B}}{\alpha}(1+n)\tilde{k}'(\cdot) = \frac{\bar{B}}{\alpha} \frac{\frac{\beta}{1+\beta}}{1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha} + \frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}} I(\cdot) \hat{y}(k,h) z(\tau), \tag{52}$$

where the second equality is derived from (42). The expression in (52) shows that the term b'/R' is linearly related to the net after-tax income, $I(\cdot) \hat{y}(k,h) z(\tau)$. An increase in τ reduces $I(\cdot) \hat{y}(k,h) z(\tau)$ as shown in (51). This, in turn, decreases the level of physical capital in the next period k': $\partial \tilde{k}'(\cdot)/\partial \tau < 0$. As a result, an increase in τ works to lower the present value of the pension benefit: $\partial [b'/R'] / \partial \tau < 0$.

The term $R\left(\tilde{k}'(\cdot), \tilde{h}'(\cdot), \bar{T}\right) = \alpha A \left(\tilde{k}'(\cdot)\right)^{\alpha-1} \left(\tilde{h}'(\cdot)\right)^{1-\alpha} z\left(\bar{T}\right)$ represents the gross interest rate. An increase in τ reduces the physical capital stock $\tilde{k}'(\cdot)$, thereby putting upward pressure on the interest rate. A rise in the interest rate, in turn, lowers the borrowing limit faced by the young, reduces their tertiary education expenditures, and consequently diminishes their human capital level in middle age. This decline in human capital works to depress the interest rate. Hence, there are two opposing effects on the interest rate, and the net impact is positive: $\partial R\left(\tilde{k}'(\cdot), \tilde{h}'(\cdot), \bar{T}\right)/\partial \tau > 0$.

Finally, consider the two terms in the denominator of (49). The first term $(1+\tau z'(\tau)/z(\tau)) \times (1-\alpha)\hat{y}(k,h) z(\tau)$ indicates that a rise in τ increases tax revenue. The second term $(-1)(1+n)\sigma\frac{\partial e}{\partial \tau}$ captures a decline in tertiary education expenditure. A rise in τ leads to a decrease in the private tertiary education expenditures of the young as explained in the previous paragraph. Therefore, the denominator in (49), which includes these two terms, represents the net increase in revenue from labor income taxation.

3.2 Fully Private Funding of Tertiary Education

In Proposition 1, we derive four policy functions—τ, σ, $(1+n)x$, and $b/(1+n)$—but we are unable to explicitly express the underlying parameters $\bar{T}$, $\bar{\Sigma}$, $\bar{X}$, and $\bar{B}$ that determine each policy function. This limitation makes it difficult to clearly identify how the two parameters of interest in this study, ϕ and n, affect the policy functions and, consequently, economic growth. To address this issue, we begin by focusing on a special case in which there is no public funding for tertiary education (i.e., $\sigma = 0$). This simplified case allows us to derive the policy functions in closed form, thereby clarifying the channels through which ϕ and n influence policy choices and economic growth. The insights gained from this special case serve as the basis for interpreting the results of the more general analysis with public funding for tertiary education, which is presented in the following section.

Proposition 2 *Under fully private funding of tertiary education ($\sigma = 0$), the policy functions in the Markov-perfect political equilibrium are given by:*

$$1-\tau = 1-\bar{T}\big|_{\sigma=0} \equiv \frac{1}{1-\alpha}\left[(1-\phi) + \frac{1}{\frac{[1+\beta(1-(1-\alpha)(1-\mu))]\left(\frac{\frac{1/v}{1+1/v}}{\frac{1/v}{1+1/v}-\phi}+v(1-\alpha)\right)}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta} + v(1-\alpha)}\right]^{-1}, \tag{53}$$

$$\frac{\frac{b}{1+n}}{\hat{y}(k,h)\,z(\tau)} = \bar{B}\big|_{\sigma=0} \equiv \frac{\frac{\omega}{(1+n)(1-\omega)}\,\bar{T}\big|_{\sigma=0}(1-\alpha) - \beta(1-\alpha)\eta\alpha\left(1+\phi\left(1-\bar{T}\big|_{\sigma=0}\right)\frac{1-\alpha}{\alpha}\right)}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}, \tag{54}$$

$$\frac{(1+n)x}{\hat{y}(k,h)\,z(\tau)} = \bar{X}\big|_{\sigma=0} \equiv \frac{\beta(1-\alpha)\eta}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}\left[\alpha\left(1+\phi\left(1-\bar{T}\big|_{\sigma=0}\right)\frac{1-\alpha}{\alpha}\right)+\bar{T}\big|_{\sigma=0}(1-\alpha)\right]. \tag{55}$$

Proof. See Appendix B.

The ϕs appeared in (53), (54), and (55) represent the borrowing constraint faced by the middle-aged in their youth. In contrast, the borrowing constraint faced by the young in the current period appears only in the second term of the denominator in the MCT of (49) and vanishes when tertiary education is fully privately funded. Therefore, the results in the following proposition capture only the effects of the borrowing constraints that the current middle-aged faced when they were young.

Proposition 3 *Under fully private funding of tertiary education ($\sigma = 0$), relaxing borrowing constraints (i.e., an increase in ϕ) has the following effects on policy variables. (i) The tax rate τ is decreasing in ϕ : $\frac{\partial \tau}{\partial \phi} < 0$; (ii) The pension expenditure-to-GDP ratio $\frac{b}{1+n}/\hat{y}(k,h)\,z(\tau)$ is decreasing in ϕ : $\frac{\partial \frac{b}{1+n}/\hat{y}(k,h)z(\tau)}{\partial \phi} < 0$; and (iii) For $v > 0$, the public non-tertiary education expenditure-to-GDP ratio $(1+n)x/\hat{y}(k,h)\,z(\tau)$ is decreasing in ϕ if $\frac{\omega}{(1+n)(1-\omega)} \le \frac{\alpha}{1-\alpha}$ ·*

$[1+\beta(1-(1-\alpha)(1-\mu))]-\beta(1-\alpha)\eta$; *increasing in* ϕ *if* $\left(\frac{1}{(1-\alpha)(1-\mu)^2}-1\right)\cdot[1+\beta(1-(1-\alpha)(1-\mu))]-\beta(1-\alpha)\eta \leq \frac{\omega}{(1+n)(1-\omega)}$; *and exhibits an inverse U-shape otherwise; for* $v=0$, *the ratio is independent of* ϕ.

Proof. See Appendix C.

To interpret the results of Proposition 3, we recall the government's objective function in (44). Under full private funding of tertiary education ($\sigma = 0$), the objective function can be written using the physical capital accumulation equation in (42) and the human capital accumulation equation in (43) as follows (for the derivation, see (A.7) in Appendix A):

$$\Omega \approx \underbrace{\frac{\omega}{(1+n)(1-\omega)}}_{(n.1)} \ln d\left(\tau, \frac{b}{1+n}, k, h, \underbrace{e_{-1}}_{(\phi.1)}\right) + [1+\beta(1-(1-\alpha)(1-\mu))] \ln I\left(\tau, k, \underbrace{e_{-1}}_{(\phi.2)}\right) \hat{y}(k,h) z(\tau) + \beta(1-\alpha)\eta \ln x. \quad (56)$$

Notice that $e_{-1} = \bar{e}(0,\tau,k) = \phi(1-\tau)\frac{1-\alpha}{\alpha}k$ is taken as given at the voting stage; changes in ϕ and n do not directly affect e_{-1} during voting. However, the young determine their tertiary education expenditures, e_{-1}, based on expectations about the labor income tax rate τ in their middle age, which itself influenced by the parameters ϕ and n. Consequently, in equilibrium, there are additional indirect effects of ϕ and n on e_{-1} that operate through their impact on the labor income tax rate τ.

The first term on the right-hand side of (56) denotes the older adults' utility, weighted by their relative political weight $\frac{\omega}{(1+n)(1-\omega)}$, and the second and third terms represent the lifetime utility of the middle-aged. In particular, the second term includes the lifetime income plus the component of the interest rate R' that captures the labor income effects via savings and private tertiary education expenditure. The third term comprises the remaining part of the interest rate, capturing the effect of public non-tertiary education expenditure x through human capital accumulation.

The terms (ϕ.1) and (ϕ.2) capture the impact of the borrowing constraint through the private tertiary education expenditure, e_{-1}, we consider here. The term (n.1) captures the effect of the population growth rate from the previous period to the current period, which we will consider next. As we see from (50), there are no direct imapcts of ϕ and n on the government budget constraint when $\sigma = 0$. Therefore, we examine here the impacts of ϕ through the terms (ϕ.1) and (ϕ.2) and the impact of n through the term (n.1), all of which appear in the political objective function in (56).

First, let us examine how the borrowing constraint ϕ affects the labor income tax rate τ. The private tertiary education expenditure, e_{-1}, is predetermined and therefore treated as given in the political decision on τ. We begin by taking τ embedded in e_{-1} as given and investigating the direct effect of ϕ on the choice of τ. We then consider the repercussion effect of τ through e_{-1}.

When ϕ increases, thus loosening the borrowing constraint, the private tertiary education expenditure in the previous period, e_{-1}, expands, as shown in (29). This expansion generates two effects through the terms (ϕ.1) and (ϕ.2). The term (ϕ.1) indicates that the increase in private tertiary education expenditure raises demand in the asset market. As a result, savings from the previous period, s_{-1}, increase to restore the asset market equilibrium. This, in turn, strengthens the adverse effect of labor income taxation on older adults' consumption, as the associated decline in labor supply reduces the interest rate. Consequently, MCT increases. The term (ϕ.2) indicates that the increase in private tertiary education expenditure, e_{-1}, reduces adjusted after-tax income $I(\tau, k, e_{-1})$ of the middle-aged, which in turn raises the marginal cost of taxation, MCT. Due to these two effects that increase MCT, a rise in ϕ leads to a decline in the labor income tax rate τ.

A decline in τ increases the private tertiary education expenditure, as $e_{-1} = \bar{e}(0, \tau, k) = \phi(1-\tau)\frac{1-\alpha}{\alpha}k$ holds in economic equilibrium (see (29)). This increase amplifies the negative effects of ϕ on τ through terms (ϕ.1) and (ϕ.2) of (56), as discussed in the preceding paragraph. The interaction between this indirect effect and the direct effect of ϕ ultimately results in a decline in the labor income tax rate τ in the political equilibrium.

Next, we examine the effect of the borrowing constraint ϕ on the pension expenditure-to-GDP ratio, $\frac{b}{1+n}/\hat{y}(k,h)z(\tau)$. When ϕ increases, thereby loosening the borrowing constraint, the private tertiary education expenditure in the previous period e_{-1} rises. As discussed above, this increase leads to higher savings in the previous period s_{-1}. Since increased saving s_{-1} reduces the need for pension benefits $\frac{b}{1+n}$ among the older adults, the marginal benefit of pension benefits $\frac{b}{1+n}$ declines. In addition, a decline in τ caused by relaxed borrowing constraint increases the private tertiary education expenditure, as $e_{-1} = \bar{e}(0, \tau, k) = \phi(1-\tau)\frac{1-\alpha}{\alpha}k$ holds in economic equilibrium, which also lowers the need for pension benefits. With this additional negative effect on pension benefits, which is included in term (ϕ.1), an increase in ϕ ultimately leads to a decline in the pension expenditure-to-GDP ratio in the political equilibrium.

Finally, we examine the effect of the borrowing constraint ϕ on the public non-tertiary education expenditure-to-GDP ratio, $(1+n)x/\hat{y}(k,h)z(\tau)$. As discussed earlier, when the borrowing constraint is relaxed, private tertiary education expenditures in the previous period e_{-1} increase. This, in turn, raises savings through the asset market equilibrium condition, leading to a decline in the marginal utility of pensions for older adults. To maintain the balance between the marginal utilities of pensions in (45) and public non-tertiary education expenditure in (46), the government reduces pensions, $b/(1+n)$, while increasing public non-tertiary education expenditure, $(1+n)x$. The larger the relative political weight, $\frac{\omega}{(1+n)(1-\omega)}$, the stronger the incentive to expand public non-tertiary education expenditure.

Conversely, a relaxation of the borrowing constraint induces the current middle-aged generation–

when young–to increase their private tertiary education expenditure; this higher investment during youth subsequently reduces their after-tax income in middle age and thereby amplifies MCT. To balance MCT in (48) with the marginal utility of public non-tertiary education expenditure in (46), the government has an incentive to lower the labor income tax rate to mitigate this effect while simultaneously reducing public non-tertiary education expenditure. The smaller the relative political weight of older adults, the greater the political influence of the middle-aged, thereby strengthening the government's incentive to reduce public non-tertiary education expenditure.

Based on these findings, we find that when the relative political weight is large, the pressure to increase $(1+n)x$ is stronger, resulting in $\partial\left[(1+n)\,x/\hat{y}\,(k,h)\,z\,(\tau)\right]/\partial\phi > 0$. In contrast, when the relative political weight is small, the tendency to reduce $(1+n)x$ dominates, leading to $\partial\left[(1+n)\,x/\hat{y}\,(k,h)\,z\,(\tau)\right]/\partial\phi < 0$. In intermediate cases, there exists a threshold level of ϕ at which the opposing forces balance, giving rise to an inverse U-shaped relationship. Notably, when $v = 0$ (i.e., labor supply is inelastic), the two opposing effects offset each other, resulting in no impact on $(1+n)x/\hat{y}\,(k,h)\,z\,(\tau)$. In other words, under this condition, the borrowing constraint does not affect the public non-tertiary education expenditure-to-GDP ratio.

Next, we consider the effects of a change in the population growth rate on the policy variables, as summarized in the following proposition.

Proposition 4 *Under fully private funding of tertiary education ($\sigma = 0$), a change in the population growth rate (n) has the following effects on policy variables. (i) The tax rate τ is decreasing in n: $\partial\tau/\partial n < 0$; (ii) the pension expenditure-to-GDP ratio $\frac{b}{1+n}/\hat{y}\,(k,h)\,z\,(\tau)$ is decreasing in n: $\partial\left(\frac{b}{1+n}/\hat{y}\,(k,h)\,z\,(\tau)\right)/\partial n < 0$; and (iii) the public non-tertiary education expenditure-to-GDP ratio $(1+n)x/\hat{y}\,(k,h)\,z\,(\tau)$ is increasing in n: $\partial(1+n)x/\hat{y}\,(k,h)\,z\,(\tau)\,/\partial n > 0$.*

Proof. See Appendix D.

A change in the population growth rate n affects the political objective function in (56) through the term (n.1). The effects of n through the terms $b/(1+n)$ and $(1+n)x$ in the political objective function and the government budget constraint are offset by adjustments in the associated policy variables, leaving the impact of the population growth rate only in the term (n.1). Accordingly, the following analysis examines how a decline in the population growth rate n, a common phenomenon in most developed countries, affects each policy variable through the term (n.1).

A decline in the population growth rate n increases the relative political weight of the older adults in the political objective function (56), as captured by the term $\frac{\omega}{(1+n)(1-\omega)}$. This shift implies a greater weight on the cost of taxation borne by the older adults through the return on their savings, which tends to reduce the tax rate τ. At the same time, it increases the weight on the benefit from pensions $b/(1+n)$, which creates an incentive to raise the level of pension benefits. Moreover, the tax rate τ also affects the consumption utility of the middle-aged. An increase in the relative political weight of the older adults necessarily implies a decrease in the relative weight of the middle-aged, thereby reducing the weight placed on the disutility from taxation experienced by the middle-aged. This, in turn, tends to raise the tax rate τ. In summary, given e_{-1}, a decline in n has two opposing effects on τ: a negative effect through the savings return channel for the older adults, and a positive effect through the reduced disutility of taxation for the middle-aged.

To capture the net effect of middle-aged individuals' consumption utility on the determination of the tax rate τ, we focus on the condition that equalizes marginal utilities (or marginal disutilities) in terms of a change in expenditure or revenue. Based on the political objective function Ω in (56), we obtain the following marginal utility equalization condition for each policy variable:

$$\frac{\partial d(\cdot)/\partial \frac{b}{1+n}}{d(\cdot)} = (-1)\frac{\frac{\partial d(\cdot)/\partial\tau}{d(\cdot)} + \left[\frac{\omega}{(1+n)(1-\omega)}\right]^{-1}\cdot\left[1+\beta\left(1-(1-\alpha)(1-\mu)\right)\right]\frac{\partial I(\cdot)\hat{y}(k,h)z(\tau)/\partial\tau}{I(\cdot)\hat{y}(k,h)z(\tau)}}{(1-\alpha)\left(1+\frac{z'(\tau)}{z(\tau)}\right)\hat{y}(k,h)z(\tau)+(-1)(1+n)\sigma\frac{\partial e}{\partial\tau}}$$
$$= \left[\frac{\omega}{(1+n)(1-\omega)}\right]^{-1}\frac{\beta(1-\alpha)\eta(1+n)}{(1+n)x}. \tag{57}$$

This condition shows that, through optimization, MCT, the marginal utility of pensions, and the marginal utility of public non-tertiary education expenditure are equalized. From the first equality, it follows that a decline in n reduces the marginal cost of taxation on the right-hand side. In other words, given e_{-1}, the net effect of a decline in n on τ is positive. Moreover, because the marginal cost of taxation decreases with a lower n, the equalization of marginal utilities and disutilities requires that the marginal benefit from pensions on the left-hand side also decrease. This leads to an increase in pension benefits.

Given e_{-1}, a decline in n increases the tax rate τ. In economic equilibrium, we have $e_{-1} = \bar{e}(0,\tau,k) = \phi(1-\tau)\frac{1-\alpha}{\alpha}k$, so an increase in the labor income tax rate τ reduces e_{-1}, which, from the perspective of the asset market equilibrium, also lowers s_{-1}. This reduction in savings increases the marginal benefit of $b/(1+n)$, thereby further raising the level of $b/(1+n)$; in other words, the increase in pension benefits is amplified. Moreover, the decline in s_{-1} reduces the marginal cost of taxation through its impact on the consumption $d(\cdot)$ of the older adults, reinforcing the upward pressure on τ. Additionally, the decrease in e_{-1} increases the disposable income of the middle-aged, which further reduces the marginal cost of taxation and amplifies the upward effect on τ. Thus, a decline in n ultimately results in a higher tax rate τ. Finally, focusing on the second equality in (57), a decrease in n lowers the (relative) weight on the marginal utility of $(1+n)x$, which leads to a reduction in the expenditure on $(1+n)x$. That is, the ratio $(1+n)x/\hat{y}(k,h)z(\tau)$ declines.

4 Quantitative Analysis

In Section 3, we derived a closed-form solution for fiscal policy functions under the assumption of fully private funding of tertiary education ($\sigma = 0$) and conducted a comparative static analysis with respect to ϕ and n. Since a closed-form solution is not attainable in the case of partial public funding ($\sigma > 0$), we address this scenario using a quantitative approach. Specifically, we calibrate the model such that its steady-state equilibrium replicates key statistics for Japan and the United States over the period 2010–2020. We focus on these two countries because they possess well-developed loan markets for private tertiary education expenditures, which markedly distinguishes them from other advanced economies (Soares, 2003; Diris and Ooghe, 2018). Table 1 summarizes the calibrated parameter values and the empirical target for calibration. Appendix E describes the sources of data used in calibration

4.1 Calibration

We take one period in the model to correspond to 30 years in the data. This assumption is standard in quantitative analyses of two- or three-period overlapping generations models (e.g., Song et al., 2012; Gonzalez-Eiras and Niepelt, 2012; Lancia and Russo, 2016). We adopt the same values for α and η in both countries. Specifically, we set $\alpha = 0.3$, a value commonly used in the literature (e.g., Gonzalez-Eiras and Niepelt, 2012). The range of estimates for η is quite broad, and the conclusions in the literature are mixed. Although somewhat dated, Card and Krueger (1996) survey the literature on the economic returns to school quality and find that 25 estimates of the elasticity of earnings with respect to per-pupil spending range from 0.01 to 0.29. In this study, we follow Blankenau and Simpson (2004), who set $\eta = 0.1$, as our framework – particularly the specification of the human capital production function – closely mirrors theirs.

For population growth rates, we use population data from the *World Population Prospects* provided by the United Nations. Let $N_{i,j}$ denote the population of country i in year j, where $i = JP$ and US denote Japan and the United States, respectively. Because one period in the model is 30 years in length, the population growth rate of country i in the model, n_i, is calculated as

$$n_i = \left(\left(\frac{N_{i,2020}}{N_{i,2010}} \right)^{1/10} \right)^{30} - 1,$$

where $(N_{i,2020}/N_{i,2010})^{1/10}$, representing the annual gross population growth rate, is 0.9980 for Japan and 1.0079 for the United States. Using this approach, we obtain population growth rates of $n_{JP} \approx -0.0583$ for Japan and $n_{US} \approx 0.2663$ for the United States.

Next, we identify the five parameters, μ_i, v_i, β_i, ϕ_i, and ω_i, by targeting the following five ratios, with the values in parentheses indicating the corresponding targets for Japan and the United States, respectively: the ratio of public non-tertiary (i.e., public primary and secondary) education expenditure to GDP, $x_{i,t}N_{i,t+1}/Y_{i,t}$, with $(0.0250, 0.0328)$; the ratio of public tertiary education expenditure to GDP, $\sigma_{i,t}e_{i,t}N_{i,t+1}/Y_{i,t}$, with $(0.0047, 0.0095)$; the ratio of private ter-

Table 1: Parameter values

Parameter	**Interpretation**	**Value**	**Empirical Target**
Exogenously calibrated parameters			
α	Physical capital share in production	0.3	Gonzalez-Eiras and Niepelt (2012)
η	Elasticity of education technology w.r.t. public expenditure	0.1	Blankenau and Simpson (2004)
n	Population growth rate	-0.0583 (JP), 0.2663 (US)	Annual gross population growth rate: 0.9980 (JP), 1.0079 (US)
A	Scale parameter in final goods production	1	Gonzalez-Eiras and Niepelt (2012)
Parameters calibrated in equilibrium			
μ	Elasticity of education technology w.r.t. private expenditure	0.057 (JP), 0.141 (US)	Public education spending (primary to post-secondary non-tertiary) / GDP: 0.0250 (JP), 0.0328 (US)
v	Frisch elasticity of labor supply	0.108 (JP), 0.091 (US)	Public spending on tertiary education / GDP: 0.0047 (JP), 0.0095 (US)
β	Discount factor	0.938 (JP), 0.617 (US)	Private spending on tertiary education / GDP: 0.0093 (JP), 0.0165 (US)
ϕ	Degree of borrowing constraint	0.024 (JP), 0.050 (US)	Public pension spending / GDP: 0.0943 (JP), 0.0697 (US)
ω	Relative political weight on older adults	0.505 (JP), 0.547 (US)	Gross domestic savings / GDP: 0.212 (JP), 0.174 (US)
ψ	Inverse weight on the disutility of labor	3.245×10^{-7} (JP), 3.872×10^{-8} (US)	Share of hours worked in total yearly hours (24×365)
D	Scale parameter in human capital production	3.960 (JP), 7.744 (US)	Annual GDP per capita growth rate: 1.005 (JP), 1.010 (US)

Note 1: JP and US denote Japan and the United States, respectively.
Note 2: The five parameters μ, ν, β, ϕ, and ω are jointly estimated by solving the five equations, targeting the ratios listed on the same lines of μ, v, β, ϕ, and ω.
Note 3: The saving-to-GDP ratio in Japan is 0.242; however, using this value as a target results in $\beta > 1$. To address this issue, we allow for a deviation of approximately 3% from the observation and conduct the calibration using 0.212 as the target value.

tiary education expenditure to GDP, $(1-\sigma_{i,t})\, e_{i,t}N_{i,t+1}/Y_{i,t}$, with $(0.0095, 0.0165)$; the ratio of public pension expenditure to GDP, $b_{i,t}N_{i,t-1}/Y_{i,t}$, with $(0.0943, 0.0697)$; and the ratio of gross domestic savings to GDP, $s_{i,t}N_{i,t}/Y_{i,t}$, with $(0.212, 0.174)$. In the present model, these five ratios include μ_i, v_i, β_i, ϕ_i, and ω_i as well as α, η, and n_i, but do not depend on ψ, A, and D. For each ratio in each country, we compute the average data for the sample period 2010-2020 and solve the system of five equations to determine μ_i, v_i, β_i, ϕ_i, and ω_i. From this approach, we obtain the parameter values for Japan as $(\mu_{JP}, v_{JP}, \beta_{JP}, \phi_{JP}, \omega_{JP}) = (0.057, 0.108, 0.938, 0.024, 0.505)$ and for the United States as $(\mu_{US}, v_{US}, \beta_{US}, \phi_{US}, \omega_{US}) = (0.141, 0.091, 0.617, 0.050, 0.547)$.

Finally, we determine ψ, A, and D for each country. First, ψ is determined using data on the proportion of total annual hours (24×365 hours) spent on labor. The average annual working hours from 2010 to 2020 were 1703 hours in Japan and 1822 hours in the United States. Accordingly, we obtain $\psi_{JP} = 3.245 \times 10^{-7}$ and $\psi_{US} = 3.872 \times 10^{-8}$. The scale parameter A in the production function is normalized to $A = 1$, following Gonzalez-Eiras and Niepelt (2012). Finally, the scale parameter D in the human capital formation function is estimated by targeting the average per middle-aged individual GDP growth rate in each country over the period 2010–2020. The average growth rates for Japan and the United States are 1.0129^{30} and 1.0146^{30}, respectively. Solving for $y'/y = 1.005^{30}$ and $y'/y = 1.010^{30}$ for D_{JP} and D_{US}, respectively (while using previously calibrated values for other parameters), we obtain $D_{JP} = 3.960$ and $D_{US} = 7.744$. For both Japan and the United States, the parameter values obtained through calibration satisfy Assumption 1.

4.2 Effects of Borrowing Constraints

We investigate the effects of the borrowing constraint ϕ on the tax rate and the public funding portion (τ and σ), as well as on the ratios of pension expenditures (bN_{-1}/Y), public non-tertiary education expenditures (xN'/Y), public tertiary education ($\sigma eN'/Y$), private tertiary education ($(1-\sigma)\, eN'/Y$), and tertiary education expenditure (eN'/Y) to GDP, and the growth rate of output per middle-aged individual (y'/y). Figures 2 and 3 plot the first seven variables and the last one, respectively, for Japan and the United States. The parameter ϕ, plotted on the horizontal axis, represents the borrowing constraint and takes values from 0 to the upper bound specified in Assumption 1. The upper bound is 0.0515 for Japan and 0.1292 for the United States.

4.2.1 Tax Rate and Ratios of Pension and Public Non-tertiary Education Expenditures to GDP

Figure 2 shows that the effects of the borrowing constraint on the tax rate (Figure 2(a)), the pension expenditure-to-GDP ratio (Figure 2(b)), and the public non-tertiary education expenditure-to-GDP ratio (Figure 2(c)) are qualitatively similar to those observed under fully private funding of tertiary education, as demonstrated in Proposition 3: relaxing the borrowing constraint (i.e., an increase in ϕ) lowers both the tax rate and the pension expenditure-to-GDP

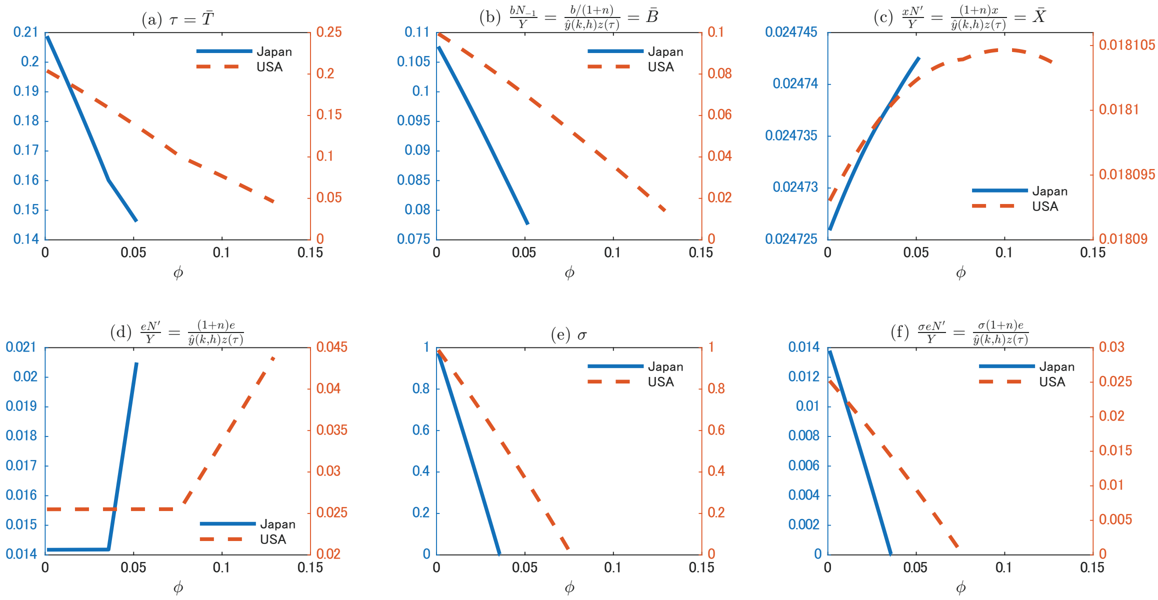


Figure 2: The effects of the borrowing constraint, ϕ, on (a) the tax rate, τ, (b) the pension-to-GDP ratio, bN_{-1}/Y, (c) the public non-tertiary education expenditure-to-GDP ratio, xN'/Y, (d) the tertiary education expenditure-to-GDP ratio, eN'/Y, (e) the public funding portion, σ, and (f) the public tertiary education expenditure-to-GDP ratio, $\sigma eN'/Y$.
Note. The left (right) vertical axis in each panel shows the corresponding values for Japan (the United States). The same convention applies to the following figures.

ratio. Japan and the United States exhibit contrasting outcomes in the public non-tertiary education expenditure-to-GDP ratio; yet, these results are consistet with Proposition 3. In Japan, relaxing the borrowing constraint leads to a monotonic increase in the ratio, whereas in the United States, it initially increases before subsequently declining. These contrasting results arise because the relative political weight of the older adults is high in Japan, whereas it is moderate in the United States.

4.2.2 Tertiary Education Expenditures

Figure 2 illustrates the effects of the borrowing constraint, ϕ, on the tertiary education expenditure-to-GDP ratio, eN'/Y, the portion of public funding for tertiary education, σ, and the public tertiary education expenditure-to-GDP ratio, $\sigma eN'/Y$. Owing to the specifications of the functions, ϕ influences the choice of σ only through its appearance in the government's budget constraint via public tertiary education expenditure, $(1+n)\sigma e(\cdot)$.

The mechanism underlying this relationship becomes clear upon examining the first term on the left-hand side of the first-order condition for σ in (47), which can be reformulated as follows:

$$\frac{\beta\frac{\partial R'/\partial\sigma}{R'}}{(1+n)\left[e(\cdot)+\sigma\frac{\partial e}{\partial\sigma}\right]}=\frac{\beta\frac{\partial R'/\partial\sigma}{R'}}{\left(1+\frac{\sigma}{1-\sigma}\right)(1+n)\,e(\cdot)}$$

$$=\frac{\overbrace{\frac{\beta(1-\alpha)\mu}{1-\sigma}}^{(*\sigma.1)}}{\underbrace{\left(1+\frac{\sigma}{1-\sigma}\right)}_{(*\sigma.2)}\cdot\underbrace{\frac{1}{1-\sigma}\underbrace{\phi}_{(*d.1)}\underbrace{\left(1-\bar{T}\right)}_{(*i.1)}\frac{1-\alpha}{\alpha}\frac{\frac{\beta}{1+\beta}\left(\frac{1/v}{1+1/v}-\underbrace{\phi}_{(*d.3)}\right)\underbrace{(1-\tau)}_{(*i.4)}(1-\alpha)\,\hat{y}(k,h)\underbrace{z(\tau)}_{(*i.5)}}{1+\underbrace{\phi}_{(*d.2)}\underbrace{\left(1-\bar{T}\right)}_{(*i.2)}\frac{1-\alpha}{\alpha}+\frac{\beta}{1+\beta}\underbrace{\frac{\bar{B}}{\beta\alpha}}_{(*i.3)}}}_{(*\sigma.3)}}, \tag{58}$$

where the equality in the first line follows from $\sigma\frac{\partial e}{\partial\sigma}=\frac{\sigma}{1-\sigma}e(\cdot)$, and the equality in the second line follows from $\frac{\partial R'/\partial\sigma}{R'}=\frac{(1-\alpha)\mu}{1-\sigma}$ and $(1+n)e=\frac{1}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}(1+n)\widetilde{k}'(\cdot)$. The term $(*\sigma.1)$ captures the marginal utility associated with increasing the public funding portion for tertiary education, σ; the term $(*\sigma.2)$ represents the portion of the resulting increase in tertiary education expenditures that is proportional to $(1+n)e(\cdot)$; and the term $(*\sigma.3)$ denotes the remaining part, corresponding to the tertiary education expenditure.

The term $(*\sigma.3)$ includes direct and indirect effects of the borrowing constraint, ϕ, on tertiary education expenditures, $(1+n)e(\cdot)$. On the one hand, the terms $(*d.1)$, $(*d.2)$, and $(*d.3)$ illustrate the direct effects of the borrowing constraint ϕ on tertiary education expenditures and, hence, the fiscal burden on the government. On the other hand, the terms $(*i.1)$, $(*i.2)$, $(*i.3)$, $(*i.4)$, and $(*i.5)$ illustrate the indirect effects operating through the labor income tax rate and pension expenditures. In particular, a change in k', stemming from the terms $(*d.2)$, $(*d.3)$, and $(*i.2)-(*i.5)$, leads to a change in $(1+n)e(\cdot)$ through the interest rate R'. This mechanism constitutes a key transmission channel in the following analysis and interpretation of the results.

We first explore the direct effects of the borrowing constraint. A relaxation of the current-period borrowing constraint increases private tertiary education spending, which in turn raises public tertiary education expenditures, thereby increasing the fiscal burden on the government (term $(*d.1)$). However, the rise in private tertiary education spending crowds out physical capital accumulation through the asset market equilibrium, leading to a rise in the future interest rate R'. This lowers the current borrowing limit, resulting in reduced private and public tertiary education expenditures, and thus a decrease in the government's fiscal cost (term $(*d.2)$). In addition, a relaxation of the borrowing constraint in the previous period reduces current-period disposable income available for savings, leading to a lower physical capital stock, a higher future interest rate, again implying a reduction in the government's fiscal burden as in $(*d.2)$ (term

$(*d.3)$). In summary, term $(*d.1)$ captures the negative effect of ϕ on the tertiary education expenditures, while terms $(*d.2)$ and $(*d.3)$ capture its positive effects.

We next turn to the indirect effects. A relaxation of the current-period borrowing constraint lowers the next-period tax rate, $\bar{T}$, which in turn increases the current private tertiary education expenditures and the corresponding public expenditures, thereby raising the fiscal burden on the government (term $(*i.1)$). In addition, an increase in private tertiary education expenditures crowds out physical capital accumulation through the asset market equilibrium, leading to a rise in the future interest rate R'. This tightens the current borrowing constraint, resulting in reduced private and public tertiary education expenditures, and thus a decrease in the government's fiscal cost (term $(*i.2)$). Meanwhile, a relaxation of the current-period borrowing constraint reduces the future pension expenditures, $\bar{B}$, which encourages saving and promotes physical capital accumulation. This, in turn, lowers the future interest rate R', thereby producing the opposite effect from that in $(*i.2)$ on the government's fiscal burden (term $(*i.3)$).

A relaxation of the borrowing constraint in the previous period also affects the current fiscal costs. It lowers the current labor income tax rate τ, increases the current savings, promotes physical capital accumulation, and thus lowers the future interest rate R'. This raises the current borrowing limit, leading to increased private and public tertiary education expenditures and a higher fiscal burden on the government (term $(*i.4)$). At the same time, the lower labor income tax rate boosts labor supply and thus output $y(\cdot) = \hat{y}(k, h)\, z(\tau)$, further promoting physical capital accumulation. The resulting decline in the future interest rate R' raises the current borrowing limit, thereby exerting the same effect on the government's fiscal cost as in $(*i.4)$ (term $(*i.5)$). In summary, term $(*i.2)$ captures the negative effect of ϕ on public tertiary education expenditures, while terms $(*i.1)$, $(*i.3)$, $(*i.4)$, and $(*i.5)$ capture its positive effects.

The numerical results indicate that the direct and indirect effects jointly exert a positive net impact on tertiary education expenditure per middle-aged individual, $(1 + n)e$, as represented by term $(*\sigma.3)$. Specifically, a relaxation of the borrowing constraint dampens physical capital accumulation (as will be shown below), which raises the interest rate R', thereby exerting a negative effect on $(1 + n)e$ as captured by terms $(*d.2)$, $(*d.3)$, and $(*i.2) - (*i.5)$. However, the same relaxation also has positive effects on $(1 + n)e$ through terms $(*d.1)$ and $(*i.1)$, which dominate the aforementioned negative effects. Consequently, the relaxation of the borrowing constraint results in a rise in $(1+n)e$, leading to an increase in the tertiary education expenditure-to-GDP ratio (see Figure 2(d)). This, in turn, makes public funding for tertiary education more costly, prompting the government to opt for a smaller share of public financing (see Figure 2(e)). As a result, the private tertiary education expenditure-to-GDP ratio, $(1 - \sigma)\, eN'/Y$, increases unambiguously. At the same time, a relaxation of the borrowing constraint entails two opposing effects on the public tertiary education expenditure-to-GDP ratio, $\sigma eN'/Y$: a positive one via the increase in the overall tertiary education expenditure-to-GDP ratio and a negative one through the reduction in σ. The numerical results indicate that the latter effect dominates, yielding a negative net outcome (see Figure 2(f)).

4.2.3 Growth Rates

The growth rate of GDP per middle-aged individual is

$$\frac{y'}{y} = \frac{\hat{y}(k', h') z(\tau')}{\hat{y}(k, h) z(\tau)} = \frac{z(\tau')}{z(\tau)} \left(\frac{k'}{k}\right)^{\alpha} \left(\frac{h'}{h}\right)^{1-\alpha}. \tag{59}$$

Equation (59) indicates that the growth rate y'/y depends on three components: the growth rates of labor supply and physical and human capital. The first component can be rewritten as

$$\frac{z(\tau')}{z(\tau)} = \left(\frac{1-\tau'}{1-\tau}\right)^{v(1-\alpha)}. \tag{60}$$

Since the tax rate depends solely on parameters, it remains constant over time, implying that the gross growth rate of labor supply equals one.

Next, consider the growth rates of physical and human capital. With the use of (42) and (43), we obtain:

$$\frac{k'}{k} = \frac{1}{1+n} \frac{Z_1}{Z_0} z(\tau) A \left(\frac{h}{k}\right)^{1-\alpha}, \tag{61}$$

$$\frac{h'}{h} = \left(\frac{1}{1+n}\right)^{\eta+\mu} Z_2 \left(\frac{Z_1}{Z_0}\right)^{\mu} (z(\tau) A)^{\eta+\mu} \left(\frac{h}{k}\right)^{-\alpha(\eta+\mu)}, \tag{62}$$

where Z_0, Z_1, and Z_2 are defined as follows:

$$\begin{aligned}
Z_0 &\equiv 1 + \phi\left(1 - \bar{T}\right) \frac{1-\alpha}{\alpha} + \frac{\beta}{1+\beta} \frac{\bar{B}}{\beta\alpha}, \\
Z_1 &\equiv \frac{\beta}{1+\beta} \left(\frac{1/v}{1+1/v} - \phi\right) (1-\tau)(1-\alpha), \\
Z_2 &\equiv D\left(\bar{X}\right)^{\eta} \left(\frac{1}{1-\sigma} \phi\left(1 - \bar{T}\right) \frac{1-\alpha}{\alpha}\right)^{\mu}.
\end{aligned}$$

We should note that ϕ appeared in Z_0 and Z_2 represents the borrowing constraint faced by the current young; ϕ appeared in Z_1 represents the borrowing constraint faced by the current middle-aged when they were young.

The term Z_0 and Z_1 come from the physical capital accumulation equation in (61). The three terms in Z_0 represent, in turn, investment for physical capital, borrowing for private tertiary education expenditure of the young, and the present value of pension benefits for the middle-aged. The term Z_1 represents the adjusted after-tax labor income of the middle-aged, devoted to saving. The term Z_2 corresponds to human capital accumulation equation in (62), in which $\bar{X}$ and $\frac{1}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}$ represent public non-tertiary education expenditures and private tertiary education expenditures, respectively.

As showin in Proposition 1, there is a unique and stable steady state of the human-to-physical capital ratio, h/k. Using (61) and (62), the steady-state growth rate, denoted by

$y'/y(= k'/k = h'/h)$ becomes:

$$\frac{y'}{y} = \left(\frac{1}{1+n}\right)^{\frac{\eta+\mu}{1-\alpha(1-\eta-\mu)}} \underbrace{\left[\frac{Z_1}{Z_0} z(\tau) A\right]^{\frac{\alpha(\eta+\mu)}{1-\alpha(1-\eta-\mu)}}}_{(*k)} \cdot \underbrace{\left[Z_2 \left(\frac{Z_1}{Z_0}\right)^{\mu} (z(\tau) A)^{\eta+\mu}\right]^{\frac{1-\alpha}{1-\alpha(1-\eta-\mu)}}}_{(*h)}, \quad (63)$$

where the terms $(*k)$ and $(*h)$ come from the growth rates of physical and human capital per middle-aged individual in (61) and (62) with the population growth effect subtracted, respectively. Figure 3 illustrates the effect of ϕ on $(Z_1/Z_0)\, z(\tau)\, A$ in $(*k)$ (Figure 3(a)), $Z_2 (Z_1/Z_0)^{\mu} (z(\tau) A)^{\eta+\mu}$ in $(*h)$ (Figure 3(b)) and the steady-state growth rate, y'/y (Figure 3(c)).

To understand the impact of borrowing constraints on steady-state growth rates, we sequentially examine the effect of ϕ on the $(*k)$ and $(*h)$ terms. The term $(Z_1/Z_0)\, z(\tau)\, A$ included in $(*k)$ captures physical capital accumulation. As discussed after (58), an increase in ϕ influences physical capital accumulation through $(*d.2)$, $(*d.3)$, $(*i.2) - (*i.5)$, with a net negative effect (see Figure 3(a)). The term $Z_2 (Z_1/Z_0)^{\mu} (z(\tau) A)^{\eta+\mu}$ included in $(*h)$ represents human capital accumulation equation, which is reformulated as follows:

$$Z_2 \left(\frac{Z_1}{Z_0}\right)^{\mu} (z(\tau) A)^{\eta+\mu} = D \left(\underbrace{\bar{X}}_{(*h.1)} \cdot \underbrace{z(\tau) A}_{(*h.2)}\right)^{\eta} \left(\underbrace{\frac{1}{1-\sigma}}_{(*h.3)} \cdot \underbrace{\phi\left(1-\bar{T}\right) \frac{1-\alpha}{\alpha} \frac{Z_1}{Z_0} z(\tau) A}_{(*h.4)}\right)^{\mu}. \quad (64)$$

The expression in (64) indicates that relaxing the borrowing constraints influences human capital accumulation through two sets of terms: $(*h.1)$ and $(*h.2)$, which represent public non-tertiary education expenditures, and $(*h.3)$ and $(*h.4)$, which represent tertiary education expenditures.

The impact through public non-tertiary education expenditures is as follows. As discussed in Section 4.2.1, public non-tertiary education expenditure increases in Japan, while in the United States, it initially rises but subsequently declines (term $(*h.1)$). In addition, the relaxation of borrowing constraints reduces the tax rate, and so encourages greater labor supply among middle-aged individuals, thereby increasing GDP per middle-aged individual. This, in turn, expands public non-tertiary education expenditures, exerting a positive effect on human capital accumulation (term $(*h.2)$).

The impact of relaxing borrowing constraints through tertiary education expenditures is as follows. Easing borrowing constraints lowers the public funding portion, leading to a decline in tertiary education expenditures. This has a negative effect on human capital accumulation (term $(*h.3)$). In addition, as discussed after (58), a relaxation of the borrowing constraint—an increase in ϕ—raises tertiary education expenditures, thereby stimulating human capital accumulation (term $(*h.4)$).

In light of these effects operating through public non-tertiary and tertiary education expenditures, we find that the net effect on human capital accumulation is positive (see Figure 3(b)). Taken together, the analysis thus far indicates that while relaxing borrowing constraints hinders physical capital accumulation, it promotes human capital accumulation. Combining these two forces, the overall impact of borrowing-constraint relaxation on GDP per middle-aged individual is positive, as illustrated in Figure 3(c).

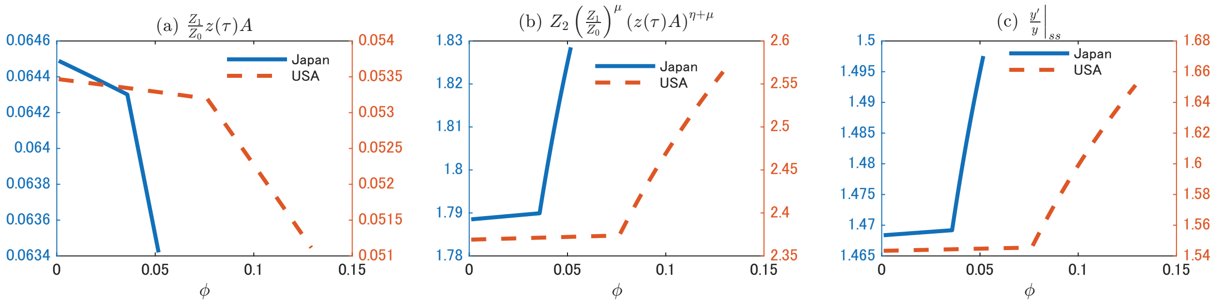


Figure 3: The effects of the borrowing constraint, ϕ, on (a) $(Z_1/Z_0)\,z(\tau)\,A$ in $(*k)$, (b) $Z_2\,(Z_1/Z_0)^{\mu}\,(z(\tau)\,A)^{\eta+\mu}$ in $(*h)$, and (c) the steady-state growth rate of GDP per middle-aged individual, y'/y.

4.3 Effects of Population Growth Rate Decline

4.3.1 Tax Rate and Ratios of Pension and Public Non-tertiary Education Expenditures to GDP

Figure 4 illustrates that the effects of a decline in the population growth rate on the tax rate, as well as on the ratios of pension and public non-tertiary education expenditures to GDP, are qualitatively similar to those observed under fully private funding of tertiary education, as demonstrated in Proposition 4. Specifically, a decrease in the population growth rate (i.e., a decline in n) increases both the tax rate (Figure 4(a)) and the pension expenditure-to-GDP ratio (Figure 4(b)) while reducing the public non-tertiary education expenditure-to-GDP ratio (Figure 4(c)). The underlying mechanism driving these results is the same as that described in Section 3.2.

4.3.2 Tertiary Education Expenditures

The population growth rate affects the determination of σ only through the term $(1+n)\,\sigma e$, which represents public tertiary education expenditures in the government budget constraint. This is analogous to the property discussed after (58) in Section 4.2.2, where the borrowing constraint ϕ influences the determination of σ only through its effect on public tertiary education expenditures in the government budget constraint. Specifically, a decrease in n raises $\bar{T}$ and $\bar{B}$, thereby generating indirect effects on physical capital accumulation that run counter to those of an increase in ϕ. As a result, a lower n impedes physical capital accumulation through terms $(*i.2)-(*i.5)$ in (58), which in turn raises the interest rate R' and thereby lowers tertiary education expenditures. At the same time, a decline in n reduces tertiary education expenditures through term $(*i.1)$. Consequently, a decline in the population growth rate leads to a decrease in the tertiary education expenditure-to-GDP ratio, eN'/Y (see Figure 4(d)).

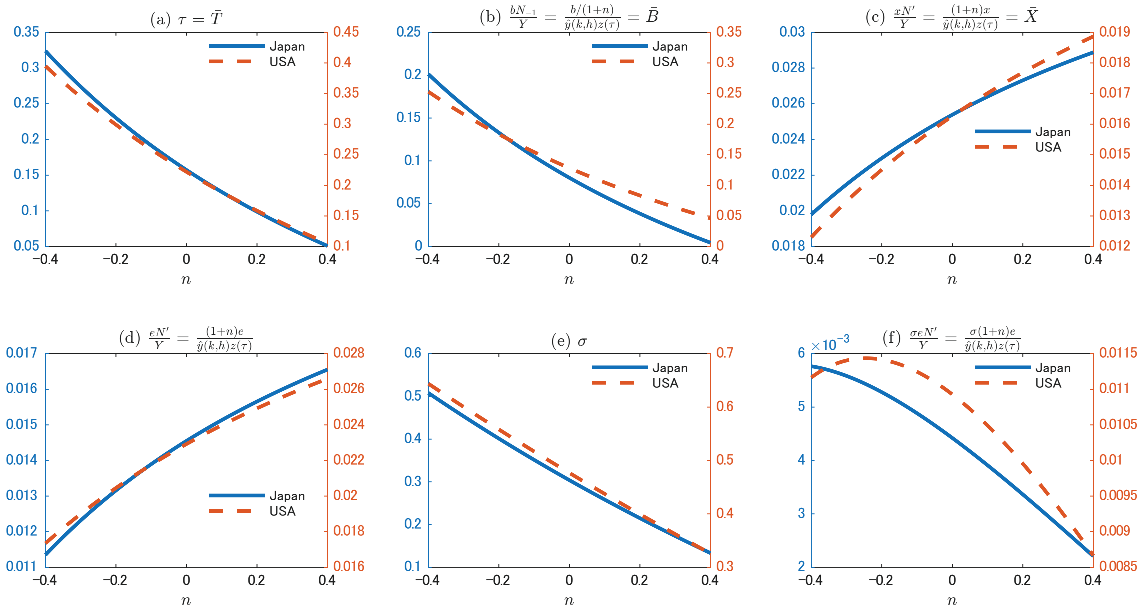


Figure 4: The effects of the population growth rate, n, on (a) the tax rate, τ, (b) the pension-to-GDP ratio, bN_{-1}/Y, (c) the public non-tertiary education expenditure-to-GDP ratio, xN'/Y, (d) the tertiary education expenditure-to-GDP ratio, eN'/Y, (e) the public funding portion, σ, and (f) the public tertiary education expenditure-to-GDP ratio, $\sigma eN'/Y$.

Given this decline in tertiary education expenditure, the fiscal burden associated with public co-funding correspondingly decreases. Thus, the government chooses to raise the portion σ (see Figure 4(e)). Consequently, the private tertiary education expenditure-to-GDP ratio, $(1-\sigma)\, eN'/Y$, decreases unambiguously. At the same time, a decline in the population growth rate exerts two opposing effects on the public tertiary education expenditure-to-GDP ratio, $\sigma eN'/Y$: a positive effect through an increase in σ and a negative effect through a decrease in the tertiary education expenditure-to-GDP ratio eN'/Y. The numerical results illustrate that Japan and the United States exhibit distinct patterns in this regard (see Figure 4(f)). In Japan, a decline in the population growth rate leads to a monotonic increase in the ratio: within the examined range of n, the positive effect of a higher σ outweighs the negative effect of a lower tertiary education expenditure-to-GDP ratio. In contrast, in the United States, there exists a level of n at which these two opposing effects offset each other, resulting in a non-monotonic pattern.

4.3.3 Growth Rates

We now examine how a decline in the population growth rate affects the growth rate of GDP per middle-aged individual, as specified in (63). A lower population growth rate influences physical capital accumulation, represented by term $(*k)$ in (63), through the four factors specified in (58), $(*i.2)-(*i.5)$. The net effect is negative (see Figure 5(a)), as discussed in Section 4.3.2. It also affects human capital accumulation, represented by term $(*h)$ in (63), through the four factors in (64): $(*h.1)-(*h.4)$.

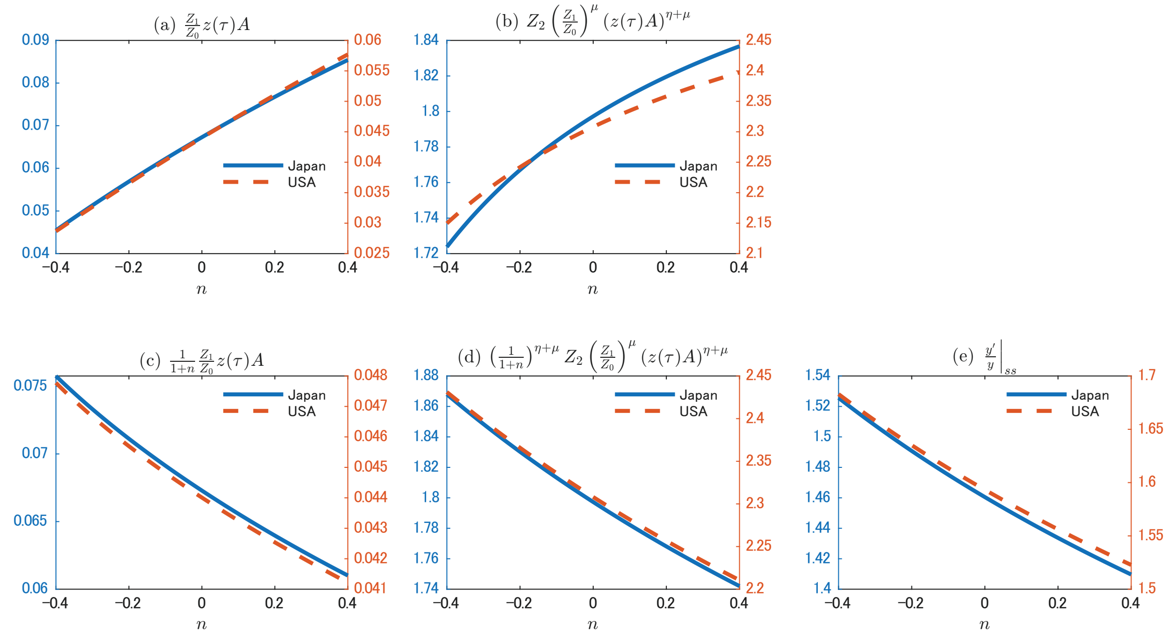


Figure 5: The effects of the population growth rate, n, on (a) $(Z_1/Z_0)\,z(\tau)\,A$ in $(*k)$ (b) $Z_2\,(Z_1/Z_0)^{\mu}\,(z(\tau)\,A)^{\eta+\mu}$ in $(*h)$, (c) $(1/(1+n))\,(Z_1/Z_0)\,z(\tau)\,A$, (d) $(1/(1+n))^{\eta+\mu}\,Z_2\,(Z_1/Z_0)^{\mu}\,(z(\tau)\,A)^{\eta+\mu}$, and (e) the steady-state growth rate of GDP per middle-aged individual, y'/y.

The details of the effects through $(*h.1)-(*h.4)$ are as follows. First, as discussed in Section 4.3.1, public non-tertiary education expenditures decrease, as captured by term $(*h.1)$. Second, since a lower population growth rate raises the tax rate, it reduces the labor supply of the middle-aged, thereby lowering GDP per middle-aged individual. This, in turn, decreases public non-tertiary education expenditures, exerting a negative effect on human capital accumulation (term $(*h.2)$). Third, lower population growth raises the share of public funding for tertiary education σ, leading to an increase in tertiary education expenditures that positively influences human capital (term $(*h.3)$). Fourth, the associated rise in the tax rate and pension expenditures discourages tertiary education expenditures (term $(*h.4)$). The net effect on human capital accumulation turns to be negative (see Figure 5(b)).

Apart from the negative effects through terms $(*k)$ and $(*h)$ in (63), a decline in the population growth rate induces capital deepening as represented by the term $(1/(1+n))^{\frac{\eta+\mu}{1-\alpha(1-\eta-\mu)}}$ in (63). In particular, the term includes the following two positive effects on the growth rate: the stimulated accumulation of physical capital through increased equipment per middle-aged individual and the stimulated accumulation of human capital through an increase in non-tertiary and tertiary education expenditures per young individual. Given these positive deepening effects on both physical and human capital, we conclude that the overall impact of a decline in the population growth rate on GDP per middle-aged individual is positive.

4.4 Analysis Based on Demographic Projections

To facilitate our analysis, we utilize a model-based time series approach to predict how fiscal policies, associated expenditures, and growth rates will evolve in response to projected changes in population growth rates. We draw on population growth data for Japan and the United States from the United Nations World Population Prospects (see Appendix E for data sources) to provide quantitative estimates of the predicted effects. In what follows, we explicitly denote the forecast year by attaching the time subscript t to each variable.

Our analysis constructs three sequences of model predictions, each spanning 30 years: (i) 2010, 2040, 2070, and 2100; (ii) 2020, 2050, and 2080; and (iii) 2030, 2060, and 2090. Since population forecasts for the 30 years following the final year of each sequence are unavailable, we assume that the population growth rate remains the same as in the preceding 30-year period. To present the time series predictions in 10-year intervals, we merge the three sequences into a single time series following the procedure outlined by Gonzalez-Eiras and Niepelt (2012).

The population projection data in Figure 6 indicate the following trends in population growth rates for Japan and the United States. In Japan, the growth rate is projected to decline continuously until 2080, with negative growth rates expected particularly after 2030. However, around 2080, while the growth rate remains negative, it is anticipated to begin recovering. In contrast, the population growth rate in the United States is projected to decline over the forecast period but remain positive throughout. Moreover, the pace of decline is expected to moderate after 2050.

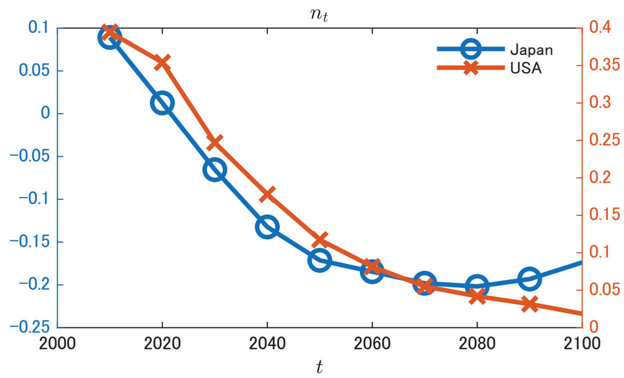


Figure 6: Projected population growth rates. The population growth rate in period t, n_t, satisfies $N_t = (1 + n_t)N_{t-1}$, where N_t denotes the population in year t.
Note. The horizontal axis in the figure reports years from 2000 in ten-year intervals; this convention also applies to the following figures.

4.4.1 Policy Variables and Associated Expenditure-to-GDP Ratios

Figure 7 presents model predictions—based on the population forecasts—for the labor income tax rate, the portion of public funding for tertiary education and the associated expenditure-to-GDP ratios. These results align with the comparative static analysis results of population growth rate decline in Section 4.3. In Japan, as the population growth rate continues to decline until 2080, both the labor income tax rate (τ_t) and the pension-to-GDP ratio ($b_t N_{t-1}/Y_t$) are projected to rise, while public non-tertiary education expenditure-to-GDP ratio ($x_t N_{t+1}/Y_t$) is projected to decrease. After 2080, when the population growth rate is expected to improve, these three policy variables are projected to move in the opposite direction. In the United States, the downward trend in population growth rate is associated with an increase in the labor income tax rate, an increase in the pension expenditure-to-GDP ratio, and a decrease in the public non-tertiary education expenditure-to-GDP ratio. Importantly, as the degree of decline in population growth rate is projected to lessen after 2050, changes in these three policy variables also become less pronounced.

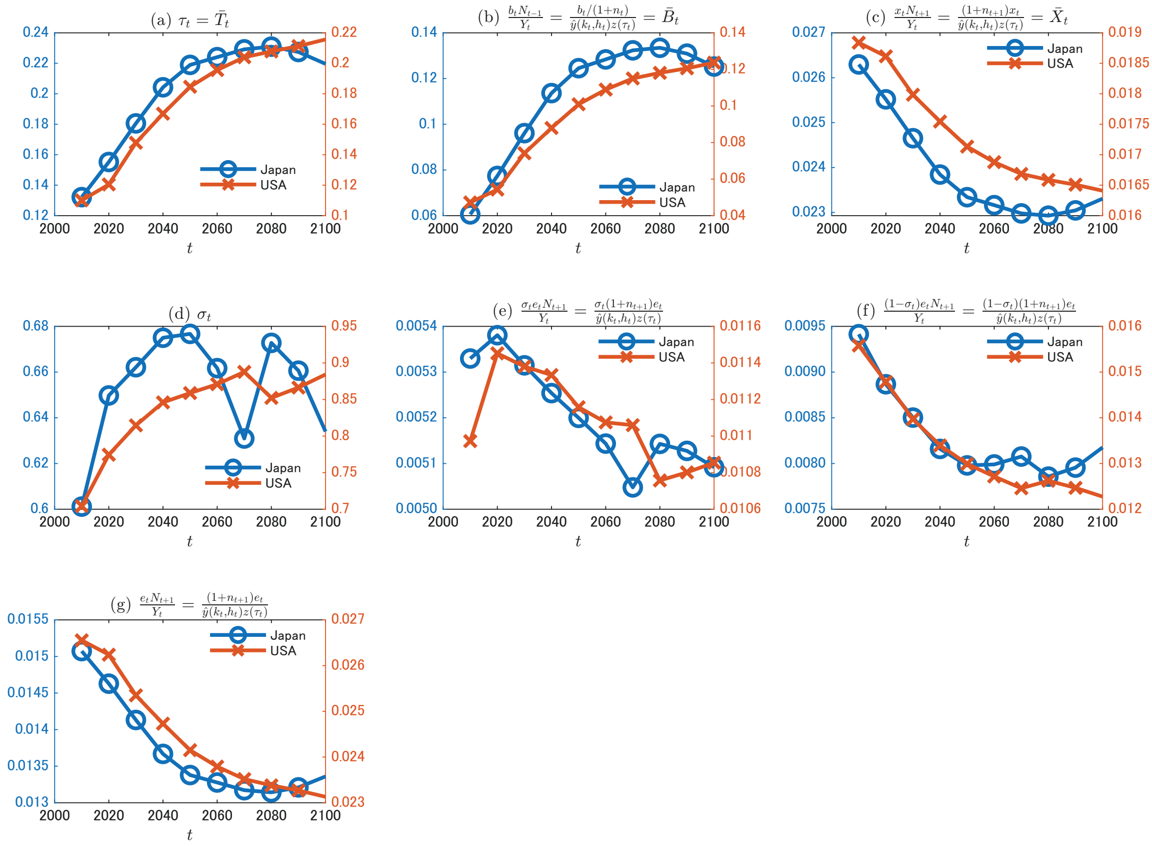


Figure 7: Model predictinos for (a) the tax rate, τ_t, (b) the pension-to-GDP ratio, $b_t N_{t-1}/Y_t$, (c) the non-tertiary education expenditure-to-GDP ratio, $x_t N_{t+1}/Y_t$, (d) the public funding portion, σ_t, (e) the public tertiary education exenditure-to-GDP ratio, $\sigma_t e_t N_{t+}/Y_t$, (f) the private tertiary education expenditure-to-GDP ratio, $(1-\sigma_t)e_t N_{t+1}/Y_t$, and (g) the tertiary education expenditure-to-GDP ratio, $e_t N_{t+1}/Y_t$.

The model's predictions for the portion of public funding in tertiary education expenditures, σ_t, the tertiary education expenditure-to-GDP ratio, $e_t N_{t+1}/Y_t$, and the corresponding public tertiary education expenditure-to-GDP ratio, $\sigma_t e_t N_{t+1}/Y_t$, are as follows. Although temporary fluctuations are observed in the public funding portion for both Japan and the United States, Japan shows that the public funding portion increases through 2080 in line with the declining population growth rate, and then declines thereafter as population growth improves. In contrast, the United States shows a generally upward trend in the public funding portion, reflecting its consistent decline in population growth rate. In addition, the tertiary education expenditure-to-GDP ratio, $e_t N_{t+1}/Y_t$, slightly decreases in both countries over the projection period. These outcomes are almost consistent with the comparative static analysis results of population growth rate decline in Section 4.3. While the public funding portion either first rises and then falls (in Japan) or rises continuously (in the United States), the public tertiary education expenditure-to-GDP ratio remains relatively stable or shows only a slight decline throughout the projection period in both countries. In other words, the effects of a declining (or increasing) population growth rate on the public tertiary education expenditure-to-GDP ratio are partially offset by the corresponding increase (or decrease) in the public funding portion, thus keeping the public tertiary education expenditure-to-GDP ratio relatively stable.

The private tertiary education expenditure-to-GDP ratio, $(1-\sigma_t)\, e_t N_{t+1}/Y_t$, remains stable or shows a slight decline throughout the projection period, similar to the public tertiary education expenditure-to-GDP ratio. The decline is more pronounced in the United States than in Japan, reflecting a stronger upward trend in the portion of public funding for tertiary education. Accordingly, the total tertiary education expenditure-to-GDP ratio, $e_t N_{t+1}/Y_t$, slightly decreases in both countries, consistent with the downward trends in its private and public components. Taken together, the results presented thus far indicate that in both countries, public spending is projected to shift from education toward pensions in response to the declining trend in population growth.

4.4.2 Growth Rates

The growth rate of GDP per middle-aged individual is expressed by (59). In the following analysis, we sequentially examine how the projected decline in population growth affects the three contributing factors to economic growth: the labor supply growth rate (which depends on the tax rate), $z(\tau_{t+1})/z(\tau_t)$, the growth rate of physical capital per middle-aged individual, k_{t+1}/k_t, and the growth rate of human capital per middle-aged individual, h_{t+1}/h_t.

Labor Supply Growth

First, we focus on the labor supply growth rate in (60). Except for Japan after 2080, both Japan and the United States are projected to experience a decline in population growth (see Figure 6), which, in turn, suggests an upward trend in labor income tax rates (seer Figure 7(a)). Consequently, a persistent decline in labor supply is expected over time, resulting in a labor supply growth rate in (60) that remains below unity. This implies that the labor supply growth rate exerts a negative effect on economic growth at each point in time. However, since the pace of increase in labor income tax rates is expected to gradually slow, the following relationship holds across periods: $\frac{1-\tau_{t+1}}{1-\tau_t} < \frac{1-\tau_{t+2}}{1-\tau_{t+1}} < \frac{1-\tau_{t+3}}{1-\tau_{t+2}} < \cdots$. As a result, the labor supply growth rate in (60) rises over time (see Figure 8(f)), and its negative impact on economic growth diminishes in later periods.

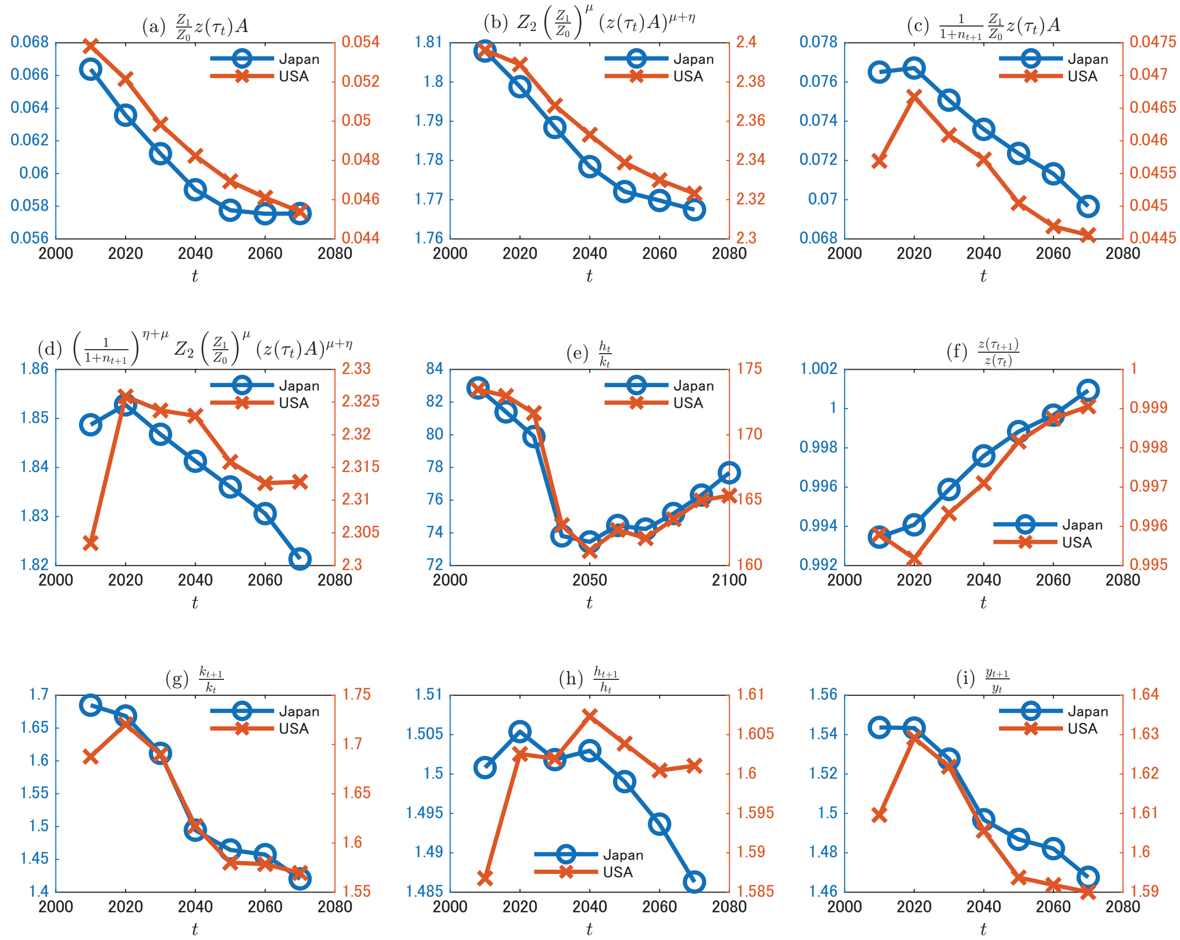


Figure 8: Model predictions for (a) $(Z_1/Z_0)\,z\,(\tau)\,A$ in $(*k)$, (b) $Z_2\,(Z_1/Z_0)^{\mu}\,(z\,(\tau)\,A)^{\eta+\mu}$ in $(*h)$, (c) $(1/\,(1+n))\,(Z_1/Z_0)\,z\,(\tau)\,A$, (d) $(1/\,(1+n))^{\eta+\mu}\,Z_2\,(Z_1/Z_0)^{\mu}\,(z\,(\tau)\,A)^{\eta+\mu}$, (e) the human-to-physical capital ratio, h_t/k_t, (f) the labor supply growth rate, $z(\tau_{t+1})/z(\tau_t)$, (g) the growth rate of physical capital per middle-aged individual, k_{t+1}/k_t, (h) the growth rate of human capital per middle-aged individual, h_{t+1}/h_t, and (i) the growth rate of GDP per middle-aged individual, y_{t+1}/y_t.

Physical Capital Accumulation

Next, we examine how the growth rate of physical capital per middle-aged individual in (61) contributes to the growth rate of GDP per middle-aged individual. Firstly, we focus on the terms $1/(1+n_{t+1})$ and $(Z_1/Z_0)\,z\,(\tau_t)\,A$ in (61), where the effects through the term $(Z_1/Z_0)\,z\,(\tau_t)\,A$ are decomposed by the terms $(*i.2)-(*i.5)$ in (58); we analyze the role of the other term h_t/k_t in (61) at a later stage. As discussed in Section 4.3.3, a decline in the population growth rate influences physical capital accumulation through the terms $(*i.2)-(*i.5)$ in (58) and the physical capital deepening factor represented by the term $1/(1+n_{t+1})$ in (61). In the analysis presented in Section 4.3, —where the population growth rate declines uniformly across all periods—the positive effects via $(*i.2)$ and $1/(1+n_{t+1})$ outweigh the negative effects via $(*i.3)-(*i.5)$. However, in the present analysis—where the population growth rate declines non-uniformly in line with demographic projections—the negative effects outweigh the positive ones, resulting in a downward trend across periods (see Figure 8(c)).

To understand this seemingly contradictory result, we highlight the key differences between the analysis in Section 4.3.3 and the present analysis (excluding the years 2090 and 2100 for Japan). In the comparative statics analysis of Section 4.3.3, the population growth rate, labor income tax rate, and pension expenditure-to-GDP ratio remain constant across periods. By contrast, the present analysis, based on demographic projections, shows that $n_t > n_{t+1}$, $\bar{T}_t < \bar{T}_{t+1}$, and $\bar{B}_t < \bar{B}_{t+1}$, indicating a declining trend in the population growth rate as well as rising trends in both the labor income tax rate and the pension expenditure-to-GDP ratio. Consequently, the present analysis amplifies the effects through $(*i.2)$ and $(*i.5)$ in (58), and weakens the capital deepening effect through the term $1/(1+n_{t+1})$ in (61). As a result, the negative effects via $(*i.3)-(*i.5)$ outweigh the positive effect via $(*i.2)$ at the net level.

In Japan, the population growth rate is projected to turn upward after 2080, with the rates in 2090 and 2100 each exceeding those of a decade earlier. Based on the preceding discussion, the term $(1/(1+n_{t+1}))\cdot(Z_1/Z_0)\,z(\tau_t)\,A$ in (61) should likewise display an increasing pattern in response to this demographic shift. However, the numerical results in Figure 8(c) show that $(1/(1+n_{t+1}))\cdot(Z_1/Z_0)\,z(\tau_t)\,A$ continues to decline. This apparent inconsistency arises from the numerical procedure, which evaluates the equilibrium at 30-year intervals, thereby integrating three independent simulations into a single sequence of results.[2]

Human Capital Accumulation

Third, we examine how the growth rate of human capital per middle-aged individual in (62) contributes to the growth rate of GDP per middle-aged individual. For analysis, we focus on term $(1/(1+n_{t+1}))^{\eta+\mu}\,Z_2\,(Z_1/Z_0)^{\mu}\,(z(\tau_t)\,A)^{\eta+\mu}$ in (62) where the term $Z_2\,(Z_1/Z_0)^{\mu}\,(z(\tau_t)\,A)^{\eta+\mu}$ is decomposed into four terms as expressed in (64): $(*h.1)-(*h.4)$, with the net effect being negative. In the analysis presented in Section 4.3, where the population growth rate declines uniformly across all periods, the positive effect through the capital deepening effect operating via $(1/(1+n_{t+1}))^{\eta+\mu}$ outweighs the negative effect via the term $Z_2\,(Z_1/Z_0)^{\mu}\,(z(\tau_t)\,A)^{\eta+\mu}$. However, in the present analysis, where the population growth rate declines non-uniformly, the negative effect outweighs the positive one, as in the case of physical capital accumulation, resulting in a downward trend across periods (see Figure 8(d)). In particular, for the United States, because the decline in the population growth rate is expected to be gradual, the decline in term $(1/(1+n_{t+1}))^{\eta+\mu}\,Z_2\,(Z_1/Z_0)^{\mu}\,(z(\tau_t)\,A)^{\eta+\mu}$ also becomes less pronounced. In contrast, for Japan, since the decrease in $(*h.4)$ does not slow, the decline in term $(1/(1+n_{t+1}))^{\eta+\mu}\,Z_2\,(Z_1/Z_0)^{\mu}$

[2] Suppose the current period is 2070. The population growth rate declines from 2040 to 2070 but rises from 2070 to 2100. The earlier decline increases the labor income tax rate in $(*i.4)$, thereby reducing $z(\tau_t)$ in $(*i.5)$. The subsequent rise in the population growth rate lowers the debt-to-GDP ratio $\bar{B}_{t+1}$ in $(*i.3)$, the labor income tax rate $\bar{T}_{t+1}$ in $(*i.2)$, and the capital deepening effect via the term $1/(1+n_{t+1})$ in (61). Consequently, the four negative effects through $(*i.2)$, $(*i.4)$, and $(*i.5)$ in (58), and the deepening effect via the term $1/(1+n_{t+1})$ in (61) outweigh the single positive effect via $(*i.3)$. This mechanism explains the continued decline in $(1/(1+n_{t+1}))\cdot(Z_1/Z_0)\,z(\tau_t)\,A$ despite the apparent upward trend in population growth at 10-year intervals. It also accounts for why the decline in $(1/(1+n_{t+1}))\cdot(Z_1/Z_0)\,z(\tau_t)\,A$ is less gradual in Japan than in the United States.

$\times (z(\tau_t) A)^{\eta+\mu}$ remains pronounced (see Figure 8(d)).

Human to Physical Capital Ratio and Growth Rate of GDP Per Middle-aged Individual

Finally, we examine how the projected decline in the population growth rate affects the human-to-physical capital ratio, and in turn, how this interacts with the growth rates of labor supply, physical capital, and human capital to influence GDP per middle-aged individual. First, let us focus on the human-to-physical capital ratio. In both Japan and the United States, this ratio initially decreases and then increases (see Figure 8(e)). Because it negatively affects physical capital accumulation but positively affects human capital accumulation, it follows a U-shaped pattern over the analysis period, initially hindering physical capital accumulation and later stimulating it (see Figure 8(g)). Conversely, its effect on human capital accumulation displays an inverse U-shaped pattern, initially promoting it and eventually constraining it (see Figure 8(h)).

From this analysis, the following summarizes the anticipated effects on economic growth in Japan and the United States, as predicted by their respective population growth rates. First, because the gross growth rate of labor supply falls below 1, it acts as a factor that worsens economic growth, although its impact gradually diminishes over time. Second, physical capital accumulation fosters economic growth; however, its contribution weakens as term $(Z_1/Z_0)\, z(\tau_t)\, A$ declines. Moreover, although the effect through term h/k is initially negative and later becomes positive, when combined with the effect of term $(Z_1/Z_0)\, z(\tau_t)\, A$, the net effect is negative. Third, human capital accumulation also promotes economic growth, but its contribution diminishes through the decline of term $Z_2 (Z_1/Z_0)^{\mu} (z(\tau_t) A)^{\eta+\mu}$ (in Japan) or its slight decrease (in the United States). Furthermore, because the effect through term h_t/k_t is initially positive and then negative, human capital accumulation as a whole has a positive impact at first, followed by a negative impact in both countries. Overall, in Japan, while the growth rate of GDP per middle-aged individual remains above 1, it follows a downward trend. In the United States, the growth rate also exceeds 1 and increases between 2010 and 2020, but subsequently declines (see Figure 8(i)). This result contrasts with the comparative statics in Section 4.3.3, which show that a uniform reduction in n raises the growth rate, thereby highlighting the importance of analyses based on population projections.

5 Conclusion

This paper has examined how borrowing constraints faced by young individuals influence the political determination of intergenerational fiscal policies and their macroeconomic consequences. Building on earlier political-economy studies of education finance, the analysis highlights that borrowing constraints play a central role in shaping the interaction between pension policies, education expenditures, and economic growth. To address this issue, the study developed a three-period overlapping-generations model with physical and human capital in which individuals finance tertiary education through credit markets while facing borrowing limits. Within this framework, the government simultaneously determines pension benefits, non-tertiary education expenditures, the public funding portion of tertiary education, and the labor income tax rate through a probabilistic voting mechanism that aggregates generational preferences. This setup allows the analysis to investigate how borrowing constraints and demographic change jointly shape the intergenerational allocation of fiscal resources and the resulting growth outcomes.

The theoretical analysis yields several key insights regarding the political determination of fiscal policy. First, the government selects policy instruments such that the marginal utility of each type of public expenditure is equalized with the marginal cost of taxation. As a result, fiscal ratios—including the pension expenditure-to-GDP ratio, the public non-tertiary education expenditure-to-GDP ratio, the public funding portion of tertiary education, and the labor income tax rate—are determined by structural parameters such as the degree of borrowing constraints and the population growth rate. A relaxation of borrowing constraints increases private tertiary education expenditures and savings, which raises the marginal cost of taxation and leads to lower labor income tax rates and reduced pension spending. The response of public non-tertiary education expenditures, however, depends on the political weight of older adults: it may increase, decrease, or display a non-monotonic pattern depending on this weight. In addition, demographic change plays a crucial role. A decline in the population growth rate increases the political influence of older adults, raising pension expenditures and the labor income tax rate while reducing public non-tertiary education spending.

The quantitative analysis, calibrated to Japan and the United States, further illustrates how borrowing constraints and demographic change interact in shaping fiscal policy and economic growth. Relaxing borrowing constraints has the same effects on the tax rate, pension expenditures, and public non-tertiary education expenditures in both countries as predicted by the theoretical analysis. In addition, with regard to the financing of tertiary education, the results indicate that the share of public funding decreases, while total expenditures increase, leading to a decline in the public tertiary education expenditure-to-GDP ratio and an increase in the private counterpart. In contrast, demographic projections indicate that declining population growth raises tax rates and pension expenditures while lowering public non-tertiary education expenditures. Moreover, although the share of public funding in tertiary education expenditures increases, total tertiary education expenditures decline. As a result, the public tertiary education expenditure-to-GDP ratio remains relatively stable or exhibits only a slight decline, reflecting offsetting effects between the rising public share and declining total expenditures, while the private tertiary education expenditure-to-GDP ratio decreases slightly.

Several limitations of the present study suggest directions for future research. First, the analysis abstracts from heterogeneity within generations, such as differences in income, ability, or access to credit markets, which may affect both education investment and political preferences over fiscal policy. Incorporating such heterogeneity would allow for a richer examination of distributional conflicts within generations and their interaction with intergenerational policy choices. Second, the model treats borrowing constraints and institutional settings of education loan markets as exogenous, whereas in reality these constraints may evolve in response to policy reforms or financial development. Endogenizing the institutional design of education finance could provide further insights into the interaction between credit market institutions and fiscal policy. Finally, extending the framework to incorporate additional policy instruments—such as capital income taxation or education subsidies targeted at specific groups—would help evaluate the broader policy mix available to governments confronting demographic aging and borrowing constraints. Addressing these issues would deepen our understanding of the political economy of education finance and intergenerational redistribution in aging economies.

A Supplementary Explanation for Section 3

A.1 Derivation of (42) and (43)

First, we derive the physical capital accumulation equation in (42). Starting from the asset market equilibrium condition in (24) and the saving function in (28), we obtain

$$\begin{aligned}(1+n)(k'+(1-\sigma)\,\bar{e}(\sigma,\tau',k')) &= \bar{s}(\tau,k,h,(1-\sigma_{-1})\,e_{-1},\tau',b',k',h') \\ &= \frac{\beta}{1+\beta}\left\{I\left(\tau,k,(1-\sigma_{-1})\,e_{-1}\right)\hat{y}\left(k,h\right)z\left(\tau\right)\right. \\ &\quad \left.-\frac{b'}{\beta\alpha\hat{y}\left(\bar{k}'\left(\cdot\right),\bar{h}'\left(\cdot\right)\right)z\left(\tau'\right)/\bar{k}'\left(\cdot\right)}\right\}. \end{aligned} \tag{A.1}$$

Substituting the conjectured policy functions (40) and (41) into (A.1) and rearranging terms yield:

$$(1+n)\left(\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}\right)k'=\frac{\beta}{1+\beta}I\left(\tau,k,(1-\sigma_{-1})\,e_{-1}\right)\hat{y}\left(k,h\right)z\left(\tau\right). \tag{A.2}$$

This expression corresponds to (42).

Next, we derive the human capital accumulation equation in (43). Combining the human capital production function in (5) with the private tertiary education expenditure function in (22), we obtain the following expression for human capital accumulation (25):

$$h'=D\left(h\right)^{1-\eta-\mu}\left(x\right)^{\eta}\left(\frac{1}{1-\sigma}\phi\left(1-\tau'\right)\frac{1-\alpha}{\alpha}k'\right)^{\mu}.$$

Substituting the conjectured tax policy function (40) gives:

$$h'=D\left(h\right)^{1-\eta-\mu}\left(x\right)^{\eta}\left(\frac{1}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}k'\right)^{\mu}. \tag{A.3}$$

This corresponds to (43).

A.2 Proof of Proposition 1

A.2.1 Proof of the First Part

The government budget constraint is given by (37). By substituting the private tertiary education expenditure function in (22), the conjectured policy function in (40), and the physical capital accumulation equation in (A.2) into (37), we obtain

$$\begin{aligned}&(1+n)x+\frac{\sigma}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\frac{\frac{\beta}{1+\beta}I\left(\tau,k,(1-\sigma_{-1})\,e_{-1}\right)}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}\hat{y}\left(k,h\right)z\left(\tau\right)+\frac{b}{1+n} \\ &=\tau\left(1-\alpha\right)\hat{y}\left(k,h\right)z\left(\tau\right). \end{aligned} \tag{A.4}$$

This corresponds to (50) in the main text.

Using the conjectured policy functions in (40) and (41), the physical capital accumulation equation in (A.2), and the human capital accumulation equation in (A.3), we can rewrite the government's objective function. First, in the indirect utility function of the middle-aged in (34), the term $I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau) + b'/R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau'\right)$ is reformulated as follows:

$$
\begin{aligned}
& I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau) + \frac{b'}{R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau'\right)} \\
& = I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau) + \frac{1}{\alpha}(1+n)\bar{B}k'; \text{ since (41)} \\
& = I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau) + \frac{\bar{B}}{\alpha} \frac{\frac{\beta}{1+\beta} I(\tau, k, (1-\sigma_{-1}) e_{-1})}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right) + \frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}} \hat{y}(k,h) z(\tau); \text{ since (40) and (A.2)} \\
& = \left[1 + \frac{\bar{B}}{\alpha} \frac{\frac{\beta}{1+\beta}}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right) + \frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}\right] I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau). \qquad \text{(A.5)}
\end{aligned}
$$

Furthermore, we have

$$
\begin{aligned}
& \beta \ln R\left(\bar{k}'(\cdot), \bar{h}'(\cdot), \tau'\right) \\
& \approx \beta(\alpha-1)\ln(1+n)k' + \beta(1-\alpha)\ln h' \\
& \approx \beta(\alpha-1)\ln(1+n)k' + \beta(1-\alpha)\ln D(h)^{1-\eta-\mu}(x)^{\eta}\left(\frac{1}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}k'\right)^{\mu}; \text{ since (A.3)} \\
& \approx \beta\left[(\alpha-1)+\mu(1-\alpha)\right]\ln(1+n)k' + \beta(1-\alpha)\eta\ln x - \beta(1-\alpha)\mu\ln(1-\sigma) \\
& \approx (-1)\beta(1-\alpha)(1-\mu)\ln\frac{\frac{\beta}{1+\beta} I(\tau, k, (1-\sigma_{-1}) e_{-1})}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right) + \frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}} \hat{y}(k,h) z(\tau) \\
& + \beta(1-\alpha)\eta\ln x - \beta(1-\alpha)\mu\ln(1-\sigma); \text{ since (A.2)} \\
& \approx (-1)\beta(1-\alpha)(1-\mu)\ln I(\tau, k, (1-\sigma_{-1}) e_{-1}) z(\tau)\hat{y}(k,h) + \beta(1-\alpha)\eta\ln x - \beta(1-\alpha)\mu\ln(1-\sigma).
\end{aligned}
$$

Hence, the indirect utility of the middle-aged in (34) can be expressed as follows:

$$
\begin{aligned}
V^M(\cdot) \approx & \left[1+\beta\left(1-(1-\alpha)(1-\mu)\right)\right]\ln I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau) \\
& + \beta(1-\alpha)\eta\ln(1+n)x - \beta(1-\alpha)\mu\ln(1-\sigma), \qquad \text{(A.6)}
\end{aligned}
$$

where the terms unrelated to voting are omitted from the expression in (A.6).

Combining the indirect utility function of the older adults in (35) with that of the middle-aged in (A.6), the government's political objective function is expressed as

$$
\begin{aligned}
\Omega(\cdot) \approx & \frac{\pi\omega}{(1+n)(1-\omega)}\ln\left(\alpha\hat{y}(k,h) z(\tau)\left(1+(1-\sigma_{-1})\frac{e_{-1}}{k}\right) + \frac{b}{1+n}\right) \\
& + \left[1+\beta\left(1-(1-\alpha)(1-\mu)\right)\right]\ln I(\tau, k, (1-\sigma_{-1}) e_{-1}) \hat{y}(k,h) z(\tau) \\
& + \beta(1-\alpha)\eta\ln(1+n)x - \beta(1-\alpha)\mu\ln(1-\sigma). \qquad \text{(A.7)}
\end{aligned}
$$

Thus, the government chooses τ, σ, $(1+n)x$, and $b/(1+n)$ to maximize the political objective function in (A.7) subject to the government budget constraint in (A.4).

The first-order conditions yield three optimality conditions and the budget constraint that jointly determine τ, σ, $(1+n)x$, and $b/(1+n)$:

$$\frac{\beta(1-\alpha)\eta}{(1+n)x} = \frac{\omega}{(1+n)(1-\omega)} \cdot \frac{1}{\alpha\left(1+(1-\sigma_{-1})\frac{e_{-1}}{k}\right)\hat{y}(k,h)z(\tau)+\frac{b}{1+n}}; \tag{A.8}$$

$$\frac{\beta(1-\alpha)\eta}{(1+n)x} = \frac{\frac{\beta(1-\alpha)\mu}{1-\sigma}}{\frac{1}{(1-\sigma)^2}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\frac{\frac{\beta}{1+\beta}}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}I\left(\tau,k,(1-\sigma_{-1})e_{-1}\right)\hat{y}(k,h)z(\tau)}; \tag{A.9}$$

$$\frac{\beta(1-\alpha)\eta}{(1+n)x} = MCT(\cdot); \tag{A.10}$$

where $MCT(\cdot)$ is defined in (49) and represents the marginal cost of taxation:

$$\begin{aligned} &MCT(\cdot)\\ &= (-1)\left\{\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{\alpha\frac{z'(\tau)}{z(\tau)}\left(1+(1-\sigma_{-1})\frac{e_{-1}}{k}\right)}{\alpha\left(1+(1-\sigma_{-1})\frac{e_{-1}}{k}\right)+\frac{b}{1+n}/\hat{y}(k,h)z(\tau)}\right.\\ &\left.+\left[1+\beta\left(1-(1-\alpha)(1-\mu)\right)\right]\frac{(-1)\frac{1/v}{1+1/v}(1-\alpha)+I\left(\tau,k,(1-\sigma_{-1})e_{-1}\right)\frac{z'(\tau)}{z(\tau)}}{I\left(\tau,k,(1-\sigma_{-1})e_{-1}\right)}\right\}\\ &\times\left\{\left(1+\tau\frac{z'(\tau)}{z(\tau)}\right)(1-\alpha)\hat{y}(k,h)z(\tau)+\Sigma_0\left(\tau,\sigma,k,(1-\sigma_{-1})e_{-1},\bar{T},\bar{B}\right)\hat{y}(k,h)z(\tau)\right\}^{-1}, \end{aligned} \tag{A.11}$$

where $\Sigma_0(\cdot)$, representing the change in tertiary education expenditure induced by a change in the tax rate, is defined as follows:

$$\begin{aligned} &\Sigma_0\left(\tau,\sigma,k,(1-\sigma_{-1})e_{-1},\bar{T},\bar{B}\right)\\ &\equiv(-1)\frac{\frac{\sigma}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\frac{\beta}{1+\beta}\left\{(-1)\frac{1/v}{1+1/v}(1-\alpha)+I\left(\tau,k,(1-\sigma_{-1})e_{-1}\right)\frac{z'(\tau)}{z(\tau)}\right\}}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}. \end{aligned} \tag{A.12}$$

Using the tertiary education expenditure function in (22), we can rewrite the optimality conditions (A.8) and (A.9) as follows:

$$\frac{\beta(1-\alpha)\eta}{(1+n)x/\hat{y}(k,h)z(\tau)} = \frac{\omega}{(1+n)(1-\omega)}\cdot\frac{1}{\alpha\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)+\frac{b}{1+n}/\hat{y}(k,h)z(\tau)}, \tag{A.13}$$

$$\frac{\beta(1-\alpha)\eta}{(1+n)x/\hat{y}(k,h)z(\tau)} = \frac{\frac{\beta(1-\alpha)\mu}{1-\sigma}}{\frac{1}{(1-\sigma)^2}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\frac{\frac{\beta}{1+\beta}\left(\frac{1/v}{1+1/v}-\phi\right)(1-\tau)(1-\alpha)}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}}. \tag{A.14}$$

Using the following expressions,

$$\frac{z'(\tau)}{z(\tau)} = (-1)\frac{v(1-\alpha)}{1-\tau};\ \text{since (16)} \tag{A.15}$$

$$(1-\sigma_{-1})e_{-1}\frac{\alpha}{k} = \phi(1-\tau)(1-\alpha);\ \text{since (22)} \tag{A.16}$$

we reformulate (A.11) as

$$MCT\left(\tau,\frac{b}{1+n}/\hat{y}\left(k,h\right)z\left(\tau\right),\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)\right)$$
$$=\frac{\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{\alpha\frac{v(1-\alpha)}{1-\tau}\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)}{\alpha\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)+\frac{b}{1+n}/\hat{y}(k,h)z(\tau)}+\left[1+\beta\left(1-\left(1-\alpha\right)\left(1-\mu\right)\right)\right]\frac{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v(1-\alpha)}{\left(\frac{1/v}{1+1/v}-\phi\right)(1-\tau)}}{\left(1-\frac{\tau}{1-\tau}v\left(1-\alpha\right)\right)\left(1-\alpha\right)\hat{y}\left(k,h\right)z\left(\tau\right)+\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)\hat{y}\left(k,h\right)z\left(\tau\right)},\tag{A.17}$$

where

$$\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)\equiv\frac{\frac{\sigma}{1-\sigma}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\frac{\beta}{1+\beta}\left[\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v\left(1-\alpha\right)\right]\left(1-\alpha\right)}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}.\tag{A.18}$$

Using (A.12), the government budget constraint in (A.4) can be rewritten as

$$\frac{(1+n)x}{\hat{y}\left(k,h\right)z\left(\tau\right)}+\frac{(-1)I\left(\tau,k,\left(1-\sigma_{-1}\right)e_{-1}\right)}{(-1)\frac{1/v}{1+1/v}\left(1-\alpha\right)+I\left(\tau,k,\left(1-\sigma_{-1}\right)e_{-1}\right)\frac{z'(\tau)}{z(\tau)}}\Sigma_0\left(\tau,\sigma,\left(1-\sigma_{-1}\right)e_{-1}\frac{\alpha}{k},\bar{T},\bar{B}\right)$$
$$+\frac{\frac{b}{1+n}}{\hat{y}\left(k,h\right)z\left(\tau\right)}=\tau\left(1-\alpha\right).\tag{A.19}$$

Furthermore, using (A.15) and (A.16), (A.19) can be written as

$$\frac{(1+n)x}{\hat{y}\left(k,h\right)z\left(\tau\right)}+\frac{\left(\frac{1/v}{1+1/v}-\phi\right)\left(1-\tau\right)}{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v\left(1-\alpha\right)}\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)+\frac{\frac{b}{1+n}}{\hat{y}\left(k,h\right)z\left(\tau\right)}=\tau\left(1-\alpha\right).\tag{A.20}$$

Thus, given $\bar{T}$ and $\bar{B}$, the four equations (A.10), (A.13), (A.14), and (A.20) jointly characterize τ, σ, $\frac{(1+n)x}{\hat{y}(k,h)z(\tau)}$, and $\frac{\frac{b}{1+n}}{\hat{y}(k,h)z(\tau)}$.

In what follows, taking $\bar{T}$ and $\bar{B}$ as given, we derive the policy functions and verify that the concjectured policy functions are correct. First, rearranging (A.13) with respect to $(b/(1+n))/\hat{y}\left(k,h\right)z\left(\tau\right)$ yields

$$\frac{\frac{b}{1+n}}{\hat{y}\left(k,h\right)z\left(\tau\right)}=\frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\left(1-\alpha\right)\eta}\cdot\frac{(1+n)x}{\hat{y}\left(k,h\right)z\left(\tau\right)}-\alpha\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right).\tag{A.21}$$

Substituting (A.21) into the government budget constraint in (A.20) gives

$$\frac{(1+n)x}{\hat{y}\left(k,h\right)z\left(\tau\right)}+\frac{\left(\frac{1/v}{1+1/v}-\phi\right)\left(1-\tau\right)}{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v\left(1-\alpha\right)}\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)+\frac{\frac{\omega}{(1+n)(1-\omega)}}{\beta\left(1-\alpha\right)\eta}\cdot\frac{(1+n)x}{\hat{y}\left(k,h\right)z\left(\tau\right)}$$
$$-\alpha\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)=\tau\left(1-\alpha\right),$$

or equivalently,

$$\frac{(1+n)x}{\hat{y}\left(k,h\right)z\left(\tau\right)}=\frac{\beta\left(1-\alpha\right)\eta}{\frac{\omega}{(1+n)(1-\omega)}+\beta\left(1-\alpha\right)\eta}$$
$$\times\left[\tau\left(1-\alpha\right)+\alpha\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)-\frac{\left(\frac{1/v}{1+1/v}-\phi\right)\left(1-\tau\right)}{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v\left(1-\alpha\right)}\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)\right].\tag{A.22}$$

This is the policy function for $(1+n)x/\hat{y}(k,h)z(\tau)$ given $\tau, \sigma, \bar{T}$, and $\bar{B}$.

Substituting (A.22) into the government budget constraint in (A.20) yields

$$\begin{aligned}
\frac{\frac{b}{1+n}}{\hat{y}(k,h)z(\tau)} &= \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta} \\
&\times \left[\tau(1-\alpha)+\alpha\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)-\frac{\left(\frac{1/v}{1+1/v}-\phi\right)(1-\tau)}{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v(1-\alpha)}\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)\right] \\
&-\alpha\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right). \qquad (A.23)
\end{aligned}$$

This is the policy function for $\frac{b}{1+n}/\hat{y}(k,h)z(\tau)$ given $\tau, \sigma, \bar{T}$, and $\bar{B}$. At this stage, note that the remaining optimality conditions that have not yet been used are (A.10) and (A.14).

The determination of τ and σ are as follows. First, substituting the policy functions (A.22) and (A.23) into (A.10) yields

$$\begin{aligned}
&\frac{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}{\tau(1-\alpha)+\alpha\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)-\frac{\left(\frac{1/v}{1+1/v}-\phi\right)(1-\tau)}{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v(1-\alpha)}\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)} \\
&= MCT\left(\tau,\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)\right)\hat{y}(k,h)z(\tau) \\
&= \frac{\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{\alpha\frac{v(1-\alpha)}{1-\tau}\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)}{\alpha\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)+\frac{b}{1+n}/\hat{y}(k,h)z(\tau)}+\left[1+\beta\left(1-(1-\alpha)(1-\mu)\right)\right]\frac{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v(1-\alpha)}{\left(\frac{1/v}{1+1/v}-\phi\right)(1-\tau)}}{\left(1-\frac{\tau}{1-\tau}v(1-\alpha)\right)(1-\alpha)+\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)}.
\end{aligned} \qquad (A.24)$$

Similarly, substituting the policy function (A.22) into (A.14) yields:

$$\begin{aligned}
&\frac{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}{\tau(1-\alpha)+\alpha\left(1+\phi(1-\tau)\frac{1-\alpha}{\alpha}\right)-\frac{\left(\frac{1/v}{1+1/v}-\phi\right)(1-\tau)}{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v(1-\alpha)}\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)} \\
&= \frac{\frac{\beta(1-\alpha)\mu}{1-\sigma}}{\frac{1}{(1-\sigma)^2}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\frac{\frac{\beta}{1+\beta}\left(\frac{1/v}{1+1/v}-\phi\right)(1-\tau)(1-\alpha)}{\left(1+\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)+\frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha}}}.
\end{aligned} \qquad (A.25)$$

Given $\bar{T}$ and $\bar{B}$, τ and σ are characterized by the two equations (A.24) and (A.25). That is, τ and σ can be written implicitly as:

$$\tau = \tau\left(\bar{T},\bar{B}\right), \sigma = \sigma\left(\bar{T},\bar{B}\right).$$

Substituting the obtained $\tau = \tau\left(\bar{T},\bar{B}\right)$ and $\sigma = \sigma\left(\bar{T},\bar{B}\right)$ into the policy function (A.23) yields

$$\begin{aligned}
\frac{\frac{b}{1+n}}{\hat{y}(k,h)z(\tau)} &= \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}\cdot\left[\tau\left(\bar{T},\bar{B}\right)(1-\alpha)+\alpha\left(1+\phi\left(1-\tau\left(\bar{T},\bar{B}\right)\right)\frac{1-\alpha}{\alpha}\right)\right. \\
&\times\left.-\frac{\left(\frac{1/v}{1+1/v}-\phi\right)\left(1-\tau\left(\bar{T},\bar{B}\right)\right)}{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v(1-\alpha)}\Sigma_0\left(\sigma\left(\bar{T},\bar{B}\right),\bar{T},\bar{B}\right)\right]-\alpha\left(1+\phi\left(1-\tau\left(\bar{T},\bar{B}\right)\right)\frac{1-\alpha}{\alpha}\right).
\end{aligned} \qquad (A.26)$$

Therefore, $\bar{T}$ and $\bar{B}$ satisfy:

$$\begin{aligned}
&\bar{T} = \tau\left(\bar{T}, \bar{B}\right), \\
&\bar{B} = \frac{\frac{\omega}{(1+n)(1-\omega)}}{\frac{\omega}{(1+n)(1-\omega)} + \beta(1-\alpha)\eta} \times \Bigg[\tau\left(\bar{T}, \bar{B}\right)(1-\alpha) + \alpha\left(1 + \phi\left(1 - \tau\left(\bar{T}, \bar{B}\right)\right)\frac{1-\alpha}{\alpha}\right) \\
&\quad - \frac{\left(\frac{1/v}{1+1/v} - \phi\right)\left(1 - \tau\left(\bar{T}, \bar{B}\right)\right)}{\frac{1/v}{1+1/v} + \left(\frac{1/v}{1+1/v} - \phi\right)v(1-\alpha)}\Sigma_0\left(\sigma\left(\bar{T}, \bar{B}\right), \bar{T}, \bar{B}\right)\Bigg] - \alpha\left(1 + \phi\left(1 - \tau\left(\bar{T}, \bar{B}\right)\right)\frac{1-\alpha}{\alpha}\right).
\end{aligned}$$

Since these two conditions are independent of k and h and depend only on parameters, it is verified that $\bar{T}$ and $\bar{B}$ are constant. In other words, the conjectured policy functions in (40) and (41) are confirmed. Accordingly, τ and $\frac{b}{1+n}/\hat{y}(k,h)z(\tau)$ are given by $\tau = \bar{T}$ and $\frac{b}{1+n}/\hat{y}(k,h)z(\tau) = \bar{B}$, where $\bar{T}$ and $\bar{B}$ are constant. Moreover, σ is constant and is given by $\sigma = \bar{\Sigma} = \sigma\left(\bar{T}, \bar{B}\right)$. Finally, from the government budget constraint in (A.20), we obtain $(1+n)x/\hat{y}(k,h)z(\tau) = \bar{X}$, where $\bar{X}$ is constant. This completes the verification of the conjecture and the derivation of the equilibrium solution.

A.2.2 Proof of the Second Part

Given the policy functions expressed by $\tau = \bar{T}$, $\sigma = \bar{\Sigma}$, $(1+n)x/\hat{y}(k,h)z(\tau) = \bar{X}$, and $\frac{b}{1+n}/\hat{y}(k,h)z(\tau) = \bar{B}$ with $(1-\sigma_{-1})e_{-1} = \phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}k$, the physical and human capital accumulation equations in (42) and (43), respectively, are reformulated as follows:

$$\frac{k'}{k} = \frac{1}{1+n}\frac{Z_1}{Z_0}z\left(\bar{T}\right)A\left(\frac{h}{k}\right)^{1-\alpha}, \tag{A.27}$$

$$\frac{h'}{h} = \left(\frac{1}{1+n}\right)^{\eta+\mu}Z_2\left(\frac{Z_1}{Z_0}\right)^{\mu}\left(z\left(\bar{T}\right)A\right)^{\eta+\mu}\left(\frac{h}{k}\right)^{-\alpha(\eta+\mu)}, \tag{A.28}$$

where Z_0, Z_1, and Z_2 are defined by

$$\begin{aligned}
Z_0 &= 1 + \frac{\beta}{1+\beta}\frac{\bar{B}}{\beta\alpha} + \phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}, \\
Z_1 &= \frac{\beta}{1+\beta}\left(\frac{1/v}{1+1/v} - \phi\right)\left(1-\bar{T}\right)(1-\alpha), \\
Z_2 &= D(\bar{X})^{\eta}\left(\frac{1}{1-\bar{\Sigma}}\phi\left(1-\bar{T}\right)\frac{1-\alpha}{\alpha}\right)^{\mu},
\end{aligned}$$

respectively. With (A.27) and (A.28), we obtain

$$\frac{h'}{k'} = \frac{\left(\frac{1}{1+n}\right)^{\eta+\mu}Z_2\left(\frac{Z_1}{Z_0}\right)^{\mu}\left(z\left(\bar{T}\right)A\right)^{\eta+\mu}}{\frac{1}{1+n}\frac{Z_1}{Z_0}z\left(\bar{T}\right)A}\left(\frac{h}{k}\right)^{\alpha(1-\eta-\mu)},$$

showing that given k_0 and h_0, $\{h/k\}$ stably converges to the unique steady state, $h/k|_{ss}$, given by

$$\left.\frac{h}{k}\right|_{ss} = \left(\frac{\left(\frac{1}{1+n}\right)^{\eta+\mu}Z_2\left(\frac{Z_1}{Z_0}\right)^{\mu}\left(z\left(\bar{T}\right)A\right)^{\eta+\mu}}{\frac{1}{1+n}\frac{Z_1}{Z_0}z\left(\bar{T}\right)A}\right)^{1/[1-\alpha(1-\eta-\mu)]}.$$

B Proof of Proposition 2

When $\sigma=0$ holds, $MCT(\cdot)$ in (A.17) becomes

$$MCT(\tau,0,\phi)=\left\{\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{\alpha\frac{v(1-\alpha)}{1-\tau}\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)}{\alpha\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)+\frac{b}{1+n}/\hat{y}\left(k,h\right)z\left(\tau\right)}\right.$$
$$\left.+\left[1+\beta\left(1-\left(1-\alpha\right)\left(1-\mu\right)\right)\right]\frac{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v\left(1-\alpha\right)}{\left(\frac{1/v}{1+1/v}-\phi\right)\left(1-\tau\right)}\right\}$$
$$\times\left\{\left(1-\tau\frac{v\left(1-\alpha\right)}{1-\tau}\right)\left(1-\alpha\right)\hat{y}\left(k,h\right)z\left(\tau\right)\right\}^{-1}. \tag{A.29}$$

Substitution of (A.29) into (A.10) leads to

$$\frac{\beta\left(1-\alpha\right)\eta}{(1+n)x}=\left\{\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{\alpha\frac{v(1-\alpha)}{1-\tau}\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)}{\alpha\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)+\frac{b}{1+n}/\hat{y}\left(k,h\right)z\left(\tau\right)}\right.$$
$$\left.+\left[1+\beta\left(1-\left(1-\alpha\right)\left(1-\mu\right)\right)\right]\frac{\frac{1/v}{1+1/v}+\left(\frac{1/v}{1+1/v}-\phi\right)v\left(1-\alpha\right)}{\left(\frac{1/v}{1+1/v}-\phi\right)\left(1-\tau\right)}\right\}$$
$$\times\left\{\left(1-\tau\frac{v\left(1-\alpha\right)}{1-\tau}\right)\left(1-\alpha\right)\hat{y}\left(k,h\right)z\left(\tau\right)\right\}^{-1}. \tag{A.30}$$

In addition, when $\sigma=0$ holds, (A.8) becomes

$$\frac{\beta\left(1-\alpha\right)\eta}{(1+n)x}=\frac{\omega}{(1+n)(1-\omega)}\cdot\frac{1/z\left(\tau\right)\hat{y}\left(k,h\right)}{\alpha\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)+\frac{b}{1+n}/\hat{y}\left(k,h\right)z\left(\tau\right)}. \tag{A.31}$$

Using (A.31) together with the government budget constraint in (A.20) under the condition $\Sigma_0\left(\sigma,\bar{T},\bar{B}\right)=0$ yields (55). Substituting (55) into (A.31) then gives (54). Finally, substituting (54) and (55) into (A.30) and simplifying yields (53).

C Proof of Proposition 3

(i) A direct computation based on (53) establishes that $\partial\tau/\partial\phi<0$.

(ii) From (54), the pension expenditure-to-GDP ratio is given by

$$\frac{b}{1+n}\frac{1}{\hat{y}\left(k,h\right)z\left(\tau\right)}=\frac{\frac{\omega}{(1+n)(1-\omega)}\tau\left(1-\alpha\right)-\beta\left(1-\alpha\right)\eta\alpha\left(1+\phi\left(1-\tau\right)\frac{1-\alpha}{\alpha}\right)}{\frac{\omega}{(1+n)(1-\omega)}+\beta\left(1-\alpha\right)\eta}.$$

From the tax rate expression in (53), we obtain $\partial\tau/\partial\phi<0$. In addition, the following relationship holds

$$\frac{\partial(-1)\phi\left(1-\tau\right)}{\partial\phi}=(-1)\frac{\partial\phi\left(1-\tau\right)}{\partial\phi}=(-1)\left[\left(1-\tau\right)-\phi\frac{\partial\tau}{\partial\phi}\right]<0;\ \text{since }\partial\tau/\partial\phi<0.$$

Therefore, it follows that

$$\frac{\partial\left[\frac{b}{1+n}\frac{1}{z(\tau)\hat{y}(k,h)}\right]}{\partial\phi}<0.$$

(iii) From Proposition 2, we obtain

$$\frac{(1+n)x}{\hat{y}(k,h)\,z(\tau)} = \frac{\beta(1-\alpha)\eta}{\frac{\omega}{(1+n)(1-\omega)} + \beta(1-\alpha)\eta}\left[1-(1-\phi)(1-\tau)(1-\alpha)\right].$$

Consider the term $(1-\phi)(1-\tau)(1-\alpha)$, which depends on ϕ. Substituting the policy function $1-\tau$ from (53) into this expression yields

$$\begin{aligned}
&(1-\phi)(1-\tau)(1-\alpha)\\
&= \frac{(1-\phi)}{\frac{\left[\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right]}{\left[\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right]v(1-\alpha)+[1+\beta(1-(1-\alpha)(1-\mu))]\left[\frac{\frac{1/v}{1+1/v}}{\frac{1/v}{1+1/v}-\phi}+v(1-\alpha)\right]}+(1-\phi)};\ \text{since (53)}\\
&= \frac{\Phi(\phi)}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}, \qquad \text{(A.32)}
\end{aligned}$$

where $\Phi(\phi)$ is defined by

$$\begin{aligned}
\Phi(\phi) \equiv (1-\phi)\Bigg\{&\left[\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right]v(1-\alpha)\\
&+[1+\beta(1-(1-\alpha)(1-\mu))]\left[\frac{\frac{1/v}{1+1/v}}{\frac{1/v}{1+1/v}-\phi}+v(1-\alpha)\right]\Bigg\}.
\end{aligned}$$

Differentiating $\Phi(\phi)$ with respect to ϕ gives

$$\begin{aligned}
\frac{\partial\Phi(\phi)}{\partial\phi} = (-1)\Bigg\{&\left[\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right]v(1-\alpha)\\
&+[1+\beta(1-(1-\alpha)(1-\mu))]\left[\frac{\frac{1/v}{1+1/v}}{\frac{1/v}{1+1/v}-\phi}+v(1-\alpha)\right]\Bigg\}\\
&+(1-\phi)[1+\beta(1-(1-\alpha)(1-\mu))]\frac{\frac{1/v}{1+1/v}}{\left(\frac{1/v}{1+1/v}-\phi\right)^2}.
\end{aligned}$$

Evaluating this derivative at $\phi=0$ yields

$$\left.\frac{\partial\Phi(\phi)}{\partial\phi}\right|_{\phi=0} = (-1)\left[\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right]v(1-\alpha)+[1+\beta(1-(1-\alpha)(1-\mu))]v\alpha.$$

Since the sign of $\partial\Phi(\phi)/\partial\phi$ is equivalent to the sign of $\partial((1+n)x/z(\tau)\hat{y}(k,h))/\partial\phi$, we have for $v>0$,

$$\begin{aligned}
&\left.\frac{\partial((1+n)x/\hat{y}(k,h)\,z(\tau))}{\partial\phi}\right|_{\phi=0} \gtrless 0\\
\Leftrightarrow\ &\frac{\omega}{(1+n)(1-\omega)} \gtrless [1+\beta(1-(1-\alpha)(1-\mu))]\frac{\alpha}{1-\alpha}-\beta(1-\alpha)\eta. \qquad \text{(A.33)}
\end{aligned}$$

Next, evaluating $\partial\Phi(\phi)/\partial\phi$ at the upper bound $\phi=\mu\frac{1/v}{1+1/v}$, we obtain

$$\begin{aligned}
\left.\frac{\partial\Phi(\phi)}{\partial\phi}\right|_{\phi=\mu\frac{1/v}{1+1/v}} &= (-1)\left[\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right]v(1-\alpha)\\
&\quad+(-1)[1+\beta(1-(1-\alpha)(1-\mu))]v\left((1-\alpha)-\frac{1}{(1-\mu)^2}\right).
\end{aligned}$$

Since the sign of $\partial\Phi(\phi)/\partial\phi$ is equivalent to the sign of $\partial((1+n)x/\hat{y}(k,h))z(\tau)/\partial\phi$, we have for $v>0$,

$$\begin{aligned}\left.\frac{\partial((1+n)x/\hat{y}(k,h)z(\tau))}{\partial\phi}\right|_{\phi=\mu\frac{1/v}{1+1/v}} &\gtrless 0\\ &\Leftrightarrow \frac{\omega}{(1+n)(1-\omega)} \gtrless [1+\beta(1-(1-\alpha)(1-\mu))]\\ &\times\left(\frac{1}{(1-\alpha)(1-\mu)^2}-1\right)-\beta(1-\alpha)\eta. \qquad \text{(A.34)}\end{aligned}$$

The conditions in (A.33) and (A.34) imply that there are two threshold values for $\frac{\pi\omega}{(1+n)(1-\omega)}$. A comparison of these thresholds shows that

$$\begin{aligned}&[1+\beta(1-(1-\alpha)(1-\mu))]\frac{\alpha}{1-\alpha}-\beta(1-\alpha)\eta\\ &<[1+\beta(1-(1-\alpha)(1-\mu))]\left(\frac{1}{(1-\alpha)(1-\mu)^2}-1\right)-\beta(1-\alpha)\eta.\end{aligned}$$

Therefore, when $v>0$, we obtain the following results:

- $(1+n)x/\hat{y}(k,h)z(\tau)$ is decreasing in ϕ if $\frac{\pi\omega}{(1+n)(1-\omega)}\le[1+\beta(1-(1-\alpha)(1-\mu))]\frac{\alpha}{1-\alpha}-\beta(1-\alpha)\eta$;
- $(1+n)x/\hat{y}(k,h)z(\tau)$ is increasing in ϕ if $[1+\beta(1-(1-\alpha)(1-\mu))]\left(\frac{1}{(1-\alpha)(1-\mu)^2}-1\right)-\beta(1-\alpha)\eta\le\frac{\omega}{(1+n)(1-\omega)}$;
- Otherwise, $(1+n)x/\hat{y}(k,h)z(\tau)$ exhibits an inverse U-shaped with respect to ϕ.

Finally, when $v=0$,

$$\Phi(\phi)|_{v=0}=[1+\beta(1-(1-\alpha)(1-\mu))],$$

which implies that $\Phi(\phi)$ is independent of ϕ. Consequently, the ratio $(1+n)x/z(\tau)\hat{y}(k,h)$ is independent of ϕ in this case.

D Proof of Proposition 4

(i) The result of $\partial\tau/\partial n<0$ is immediate from (53).

(ii) The pension expenditure-to-GDP ratio, $\bar{B}|_{\sigma=0}$, in (54), is reformulated as follows:

$$\bar{B}|_{\sigma=0}=\tau(1-\alpha)-\frac{\beta(1-\alpha)\eta}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}[1-(1-\phi)(1-\tau)(1-\alpha)], \qquad \text{(A.35)}$$

where the term $1-(1-\phi)(1-\tau)(1-\alpha)$ is reformulated, using (53), as

$$1-(1-\phi)(1-\tau)(1-\alpha)=\frac{(1-\tau)(1-\alpha)}{\frac{[1+\beta(1-(1-\alpha)(1-\mu))]\left[\frac{\frac{1/v}{1+1/v}}{\frac{1/v}{1+1/v}-\phi}+v(1-\alpha)\right]}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}+v(1-\alpha)}. \qquad \text{(A.36)}$$

Substitution of (A.36) into (A.35) leads to

$$\begin{aligned}\bar{B}\big|_{\sigma=0} &= \tau(1-\alpha) - \beta(1-\alpha)\eta(1-\tau)(1-\alpha) \\ &\times\left\{[1+\beta(1-(1-\alpha)(1-\mu))]\left[\frac{\frac{1/v}{1+1/v}}{\frac{1/v}{1+1/v}-\phi}+v(1-\alpha)\right]\right. \\ &\left.+v(1-\alpha)\left(\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right)\right\}. \end{aligned} \quad \text{(A.37)}$$

Given $\partial\tau/\partial n < 0$, (A.37) shows that $\bar{B}\big|_{\sigma=0}$ is decreasing in n.

(iii) The non-tertiary education expenditure-to-GDP ratio, $\bar{X}\big|_{\sigma=0}$, in (55) is reformulated as follows:

$$\bar{X}\big|_{\sigma=0} = \frac{\beta(1-\alpha)\eta}{\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta}\left[1-(1-\phi)(1-\tau)(1-\alpha)\right],$$

which is further reformulated, using (A.36), as follows:

$$\bar{X}\big|_{\sigma=0} = \frac{\beta(1-\alpha)\eta(1-\tau)(1-\alpha)}{[1+\beta(1-(1-\alpha)(1-\mu))]\left[\frac{\frac{1/v}{1+1/v}}{\frac{1/v}{1+1/v}-\phi}+v(1-\alpha)\right]+v(1-\alpha)\left(\frac{\omega}{(1+n)(1-\omega)}+\beta(1-\alpha)\eta\right)}. \quad \text{(A.38)}$$

Given $\partial\tau/\partial n < 0$, (A.38) shows that $\bar{X}\big|_{\sigma=0}$ is increasing in n.

E Data Sources for Calibration

We use the following data for calibration.

- The data on the ratio of public spending on primary and post-secondary non-tertiary education to GDP, corresponding to the ratio $(1+n)x/z(\tau)\hat{y}(k,h)$ in the model, are sourced from OECD Public spending on education (indicator) (https://doi.org/10.1787/f99b45d0-en) (accessed on February 9, 2024). Public spending on education includes both direct expenditures on educational institutions and education-related public subsidies provided to households and administered through educational institutions.

- The data on the ratio of public spending on tertiary education to GDP, which corresponds to the ratio $\sigma e/\hat{y}(k,h)z(\tau)$ in the model, are also sourced from OECD Public Spending on Education (indicator) (https://doi.org/10.1787/f99b45d0-en) (accessed on February 9, 2024).

- The data on the ratio of private spending on tertiary education to GDP, corresponding to the ratio $(1-\sigma)e/\hat{y}(k,h)z(\tau)$ in the model, are sourced from OECD Private Spending on Education (indicator) (https://doi.org/10.1787/6e70bede-en) (accessed on February 9, 2024). Private spending on education refers to expenditures financed by private sources, including households and other private entities.

- The data on the ratio of pension spending to GDP, corresponding to the ratio $\frac{b}{1+n}/\hat{y}(k,h)z(\tau)$ in the model, are sourced from OECD Pension Spending (indicator) (https://doi.org/10.1787/a041f4ef-en) (accessed on January 05, 2024). Pension spending includes all cash expenditures on old-age and survivors' pensions, including lump-sum payments.

- The data on the ratio of gross domestic savings to GDP, corresponding to the ratio $s/\hat{y}(k,h)z(\tau)$ in the model, are sourced from the World Bank World Development Indicators (https://databank.worldbank.org/source/world-development-indicators) (accessed on January 9, 2024). Gross domestic savings are calculated as GDP minus final consumption expenditure.

- The data on average working hours are sourced from OECD Hours Worked (indicator) https://doi.org/10.1787/47be1c78-en) (accessed on December 31, 2024). Average annual hours worked are defined as the total number of hours actually worked per year divided by the average number of employed persons per year.

- The data on the average population growth rate are sourced from the United Nations World Population Prospects, Online Edition (https://population.un.org/wpp/) (accessed on January 9, 2024).

- We define GDP per middle-aged individual as GDP divided by the total population aged 15–64. The data on GDP and the total population aged 15–64 are obtained from the World Bank World Development Indicators (https://databank.worldbank.org/source/world-development-indicators) (accessed on July 29, 2024).

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